MTH 212 - MULTIVARIATE CALCULUS - PRACTICE PROBLEMS FOR EXAM III

Please be aware that this is not intended as a comprehensive list of all possible problem types!

1. Evaluate $\displaystyle\lim_{(x,y)\to(1,0)} \frac{\sin y + \ln x}{x^2 + y^2}$.

2. Show that the function $f(x,y) = \dfrac{(x-1)^2 + y^2}{(x-1)^2 + 2y^2}$ has no limit as (x,y) approaches $(1,0)$.

3. Evaluate the limit or show that the limit does not exist. Show proper work.

   (a) $\displaystyle\lim_{(x,y)\to(0,\frac{\pi}{2})} \frac{\sin y \cos y}{y e^x}$.

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y^2}{x^4 + 3y^4}$.

4. Let $f(x,y,z) = x^2 y + z^2 + 8$ and $P = (2,1,0)$. Find the unit vector which points in the direction of most rapid increase of $f(x,y,z)$ at P.

5. Suppose $\nabla f = \langle 3x^2 + 4y, \frac{5}{2}x + 16y\rangle$ for some function $f(x,y)$ and let P be the point $(2,0)$.

   (a) Compute the direction of greatest decrease at P (be sure it's a unit vector!)

   (b) Compute the directional derivative of f at P in the direction found in part (a).

   (c) Find all directions at which the directional derivative at P is 0.

6. Let $f(x,y) = \cos x + \sin y$. Give an equation for the plane tangent to the graph of $z = f(x,y)$ at $P(\frac{\pi}{2}, \pi, 0)$. Simplify your solution to the form $Ax + By + Cz = D$.

7. Consider the surface $F(x,y,z) = xy^2 z + \ln(xy) - 1 = 0$. Find the equation of the tangent plane at the point $(e,1,0)$. Simplify your solution to the form $Ax + By + Cz = D$.

8. Consider the function $F(x,y,z) = \sin x + \cos y + e^z$.

   (a) Evaluate ∇F at the point $P(\pi, 0, \ln 2)$.

   (b) Find parametric equations for the normal line to the level surface $F(x,y,z) = 1$ at P.

9. Give the linearization of the function $f(x,y) = e^x + \sin y$ at the point $P = (0,\pi)$.

10. Give the linearization of the function $g(x,y) = x^2 y + y$ at the point $P = (1,2)$.

11. (a) For a function $f(x,y,z)$ with $x = x(r,t)$, $y = y(r,t)$, and $z = z(r,t)$, what is the general multivariate chain rule for computing $\partial f/\partial r$?

    (b) Let $f(x,y,z) = x^2 + 2yz + 3z^2$, $x(r,t) = r^2 + \cos(\ln t)$, $y(r,t) = e^{t-1}$, and $z(r,t) = r^2 + t^2$. Evaluate $\partial f/\partial r$ at the point $(r,t) = (1,1)$.

12. (a) For a function $f(r,s)$ with $r = r(x,y,z)$ and $s = s(x,y,z)$, what is the general multivariate chain rule for computing $\partial f/\partial y$?

(b) Let $f(r,s) = rs + s^2$ with $r = xy^2$ and $s = \cos(yz)$. Evaluate $\partial f/\partial y$ at the point $(x,y,z) = (e, \pi/2, 4)$.

13. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $x^3z^2 - 5xy^5z = x^2 + y^3$

14. Find (but DO NOT classify) all critical points of $f(x,y) = x^2 - \frac{1}{18}(x-1)y^2$.

15. The function $g(x,y) = x^3 + x^2 + 3y^2 - x - 12y + 11$ has critical points $(-1,2)$ and $(\frac{1}{3}, 2)$. Classify these two points as saddle points and local extrema (max/min). Show proper work, computing any necessary values for classification.

16. Consider the function $f(x,y) = x^3 + 3x^2 + y^2 + 2y$. Fill in the four empty boxes in the following table. Each row corresponds to one of the two critical points. The columns provide the point, the value $f_{xx}f_{yy} - f_{xy}^2$ at the point, and the interpretation of that value (local min/max or saddle point).

Point	$f_{xx}f_{yy} - f_{xy}^2$	Min/Max/Saddle
$(0,-1)$		
	-12	

17. Consider the function $f(x,y) = x^3 + 8y^3 - 12xy$.

(a) $(0,0)$ is a critical point. Classify it as a min, a max, or a saddle point. Clearly identify the value of $f_{xx}f_{yy} - f_{xy}^2$ and f_{xx} to distinguish between a max and a min (if needed).

(b) Find and classify all remaining critical points, clearly ientifying values of $f_{xx}f_{yy} - f_{xy}^2$ and f_{xx} when necessary.

18. Suppose we want to find the point on the plane $x + 2y + 3z = 4$ closest to the point $(1,1,2)$. Write the system of equations you would need to solve to use Lagrange multipliers for this problem, but DO NOT solve the system.

19. Using Lagrange multipliers, find the point on the line $x + 2y = 5$ nearest to the origin.