Homework 5 due November 7th

Numerical Analysis, Fall 2016

Please show your work. For the programming exercises hand in your Matlab codes and outputs. (You can use the matlab function *diary* to copy your command window output.) Read the assignments carefully, make your solutions clear to read, give explanations to your answers, label axes and graphs.

1. Let $f(x) = \sin x$ and use the forward difference $f'(x) \approx (f(x+h) - f(x))/h$ to approximate $f'(\pi/6) = \sqrt{3}/2$. In MATLAB, produce a table with columns h, $(\sin(\pi/6+h) - \sin(\pi/6))/h$, and the error of approximation, $|(\sin(\pi/6 + h) - \sin(\pi/6))/h - \cos(\pi/6)|$, for $h = 10^{-k}$, $k = 1, 2, \ldots, 16$. You can use *fprintf* with appropriate options to display 15 places in your error values. Draw a log-log scale plot of the error vs h. (Recall: the best accuracy is achieved when $h \approx \sqrt{\epsilon_{machine}} \approx 10^{-8}$.)

2. Let $f(x) = \sin x$ and use the centered difference formula $f'(x) \approx (f(x+h) - f(x-h))/(2h)$ to approximate $f'(\pi/6) = \sqrt{3}/2$. In MATLAB, produce a table with columns h, $(\sin(\pi/6 + h) - \sin(\pi/6 - h))/(2h)$, and the error of approximation, $|(\sin(\pi/6+h) - \sin(\pi/6-h))/(2h) - \cos(\pi/6)|$, for $h = 10^{-k}$, $k = 1, 2, \ldots, 16$. Draw a log-log scale plot of the error vs h. (Recall: the best accuracy is achieved when $h \approx \sqrt[3]{\epsilon_{machine}} \approx 10^{-5}$.)

3. Use the same function as in (1) and (2) and approximate $f''(\pi/6) = -\sin(\pi/6) = -\sqrt{3}/2$ using $f''(x) \approx (f(x+h) - 2f(x) + f(x-h))/(h^2)$ and the same range for h. Draw a log-log scale plot of the error vs h. We expect to see that the smallest error will occur when $h \approx \sqrt[4]{\epsilon_{machine}} \approx 10^{-4}$.

4. Using *chebfun*.

   Define the Runge function in *chebfun* by typing

   ```
   f = chebfun('1./(1+x.^2)',[-5,5])
   ```

   Next, differentiate f by typing:

   ```
   fp = diff(f)
   fpp = diff(f,2) % we can differentiate up to any order k: diff(f,k)
   ```

Check the length of f, fp, and fpp to see that f is represented by a polynomial of degree 182, and fp and fp are represented by polynomials of degree 181 and 180, respectively. (The kth derivative has degree $182 - k$.)

Plot f, fp, and fpp in one fugure, use *legend* to annotate the graph.

5. Use *chebfun* to evaluate $f''(x)$ for $f(x) = \sin x$ and $x = \pi/6$. What is the degree of the interpolation polynomial that it produces for f and f'', and what is the error in its approximation to $f''(\pi/6)$?

6. Chapter 9, Exercises 2 (use MATLAB) and 5.