Lab 3, February 24th, 2016
Numerical Linear Algebra, Spring 2016

1. Compute e^x using the Taylor series expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

and the following Matlab code:

```
% Compute exp(x) using the Taylor series expansion.
% This code works fine for x>0 (try it).
% Now pick x<0
x = -40; % choose x value
oldsum = 0;
newsum = 1;
term = 1;
n = 0;
while newsum ~= oldsum,
    n = n+1;
    term = term*x/n; % x^n/n!
    oldsum = newsum;
    newsum = newsum + term;
end
fprintf('My result is %s\n', newsum);
fprintf('Compare to the correct one: %s\n', exp(x));
```

The code works fine for $x > 0$ (check it). Now compute the result for $x = -20$ and compare it to the correct result using Matlab function $exp(x)$. Note that the problem is well-conditioned. (By the way, can you find the absolute and relative condition numbers?) The issue is in the algorithm, it's *unstable*.

Try to modify the above code to produce accurate results when x is negative. (*Hint:* $e^{-x} = 1/e^x$.)

2. Using Matlab functions *poly* and *polyder*, compute the relative condition numbers of the roots $x_1, x_2, \ldots, x_{20}$ of the Wilkinson polynomial $w(x) = (x-1)(x-2)\ldots(x-20)$ with respect to perturbation of the coefficient of the term x^{19}, a_{19}, from -210 to, for instance, $-210 + x^{-23}$. You can store your results in a 20-dimensional vector. Observe that some roots are more ill-conditioned than the others. (*Recall:* condition number for the jth root is $\kappa(x_j) = \dfrac{|a_{19} x_j^{18}|}{|w'(x_j)|}$.)