Homework 2: due February 12
MTH 365/465 Numerical Linear Algebra, Spring 2018

Note: For computational problems, you should attach both your code(s) and output.

1. (15pts) Chapter 6, Exercise 1 (a,b,c).

2. (10pts) Chapter 6, Exercise 2.

3. (15pts) *Numerical differentiaton.* One can aproximate the derivative of a function $f(x)$ by
$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$
Since, using Taylor theorem with reminder,
$$f(x+h) = f(x) + hf'(x) + (h^2/2)f''(\xi), \quad \xi \in [x, x+h],$$
the *truncation error* in approximation of $f'(x)$ is $O(h)$:
$$\left| f'(x) - \frac{f(x+h) - f(x)}{h} \right| = (h/2)f''(\xi).$$
   (a) (5pts) Let $f(x) = \sin(x)$ and $x = \pi/3$. Is the problem of computing $f'(x) = \cos(x)$ well-conditioned or ill-conditioned?

   (b) (10pts) Use $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$ to approximate $f'(x)$ for $f(x) = \sin(x)$ and $x = \pi/3$. Take $h = 1.e{-}1, 1.e{-}2, \ldots, 1e{-}16$ and make a table of your results and the difference between the computed results and the true value of $f'(x)$, namely, $\cos(\pi/3) = 0.5$. (You may use MATLAB hints in Section 2.9 to create a table.) Note that the smallest error is achieved when $h \approx \sqrt{\epsilon_{machine}}$ (you do not have to show this), so you should see this in your table. Write your conclusions. (Recall: typing *eps* in MATLAB gives double-precision $\epsilon_{machine}$ by default.)

4. (15pts) Using MATLAB functions *poly* and *polyder*, compute the relative condition numbers of the roots $x_1, x_2, \ldots, x_{20}$ of the Wilkinson polynomial
$w(x) = (x-1)(x-2)\cdots(x-20) = x^{20} - 210x^{19} + \ldots$ with respect to perturbation of

the coefficient of x^{19}, a_{19}, from -210 to $-210 + 2^{-23}$. Present your results in tabular form and write your conclusion on the ill-conditioninig of the Wilkinson polynomial. (*Recall:* condition number for the jth root is $\kappa(x_j) = \dfrac{|a_i x_j^{i-1}|}{|w'(x_j)|}$.)

5. (5pts each) **MTH 465 students only:** Chapter 6, Exercises 1(f) and 3.