Homework 5: due March 28th
MTH 365/465 Numerical Linear Algebra, Spring 2018

1. (10pts) Find a line of best fit for the data set in the table. Set up the equations and then the matrix form of the system. Solve the system by any method (Normal Equations, SVD, or QR factorization) - by hand or using MATLAB. Show your work clearly. Finally, provide a plot of the data and the fitted line (in the same figure).

x	y
5	28
10	39
15	48
20	65
25	72
30	82

2. (15pts) Chapter 7, Problem 14.

3. (15pts) Find best least squares solution to the following linear system using QR factorization by writing a MATLAB code based on the pseudocode for QR provided in class:

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 4 & 6 \\ 1 & 1 & 0 \\ 1 & 4 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 2 \\ 2 \end{pmatrix}.$$

   Hand in your code and outputs.

4. (25pts) **Note:** in this exercise you can use MATLAB to calculate matrix multiplications or find eigenvalues or eigenvectors, but not to calculate the SVD directly.

Consider the matrix

$$A = \begin{pmatrix} -2 & 11 \\ -10 & 5 \end{pmatrix}.$$

(a) (10pts) Determine, by hand, an SVD of A, $A = U\Sigma V^T$. The SVD is not unique, so find the one with the minimum number of minus signs in U and V. List the singular values σ_1 and σ_2, and the left and right singular vectors.

(b) (5pts) Find A^{-1} not directly, but via the SVD. Check your result by typing *inv(A)* in MATLAB or calculating A^{-1} by other methods.

(c) (10pts) Find the eigenvalues λ_1 and λ_2 of A. Verify that $\det A = \lambda_1\lambda_2$ and $|\det A| = \sigma_1\sigma_2$.

5. (10pts) **MTH 465 students only:** Prove that the absolute value of the determinant of a square matrix is the product of its singular values.