MTH 365/465 STUDY GUIDE FOR EXAM II SPRING 2016

Exam II will be held on Friday, April 1st, 12-1 pm, in room 403. You can use a calculator and a two-sided letter size cheat-sheet. We will have a review session during our Wednesday class, on March 30th.

What you need to know for the second exam:

- Conditioning of problems, stability of algorithms (Chapter 6):
  - well- and ill-conditioned problems, the absolute and relative condition numbers, backward stability.

- Conditioning of linear systems (Chapter 7, section 7.4):
  - Vector norms (definitions, geometric interpertation, and computation), vector norms inequalities (for inequalities, give examples of non-zero vectors for which equalities are achieved).
  - General definition of a matrix norm, matrix norms induced by vector norms, Frobenius norm ($||A||_F = \sqrt{\sum_{i,j=1}^{n} a_{ij}^2} = \sqrt{trace(A^T A)} = \sqrt{trace(AA^T)} = \sqrt{\sum_{i=1}^{n} \sigma_i^2}$, for $A_{n\times n}$). Compute matrix norms by hand.
  - The condition number $\kappa(A)$ of a nonsingular matrix A. Use $\kappa(A)$ to define if A is well- or ill-conditioned.

- Iterative methods for solving $A\mathbf{x} = \mathbf{b}$ (Chapter 12, section 12.2.1 - 12.2.3):
  - Simple iteration and preconditioners.
  - Matrix splitting and matrix form of the simple iteration.
  - Algorithms based on the simple iteration:
    Jacobi iteration, the Gauss-Seidel method, the SOR method.
  - Spectral radius of a matrix A, $\rho(A)$. Show that $\rho(A) \leq ||A||$ for any induced matrix norm $||\cdot||$. Convergence of the simple iteration ($\rho(I - M^{-1}A) < 1$).
  - Convergence of Jacobi iteration for strictly diagonally dominant matrices. Convergence of the Gauss-Seidel method for symmetric and positive definite matrices.