

Numerical Linear Algebra, Spring 2016
FINAL EXAM
Due Thursday May 12th, 2016

Instructions: This is a take-home exam. You may *not* discuss the exam problems with anyone but me, the work should be yours only. Please write your solutions on your own paper. Start a new page for each problem.

1. (25pts) Write a Matlab code to implement the *inverse iteration with shift* for the matrix A given below. Use your routine to compute all the eigenvalues and corresponding eigenvectors of A by choosing an appropriate shift s each time. (You may use the algorithm given on page 314 of the textbook.) Submit your code and results.

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 5 & 1 \\ -2 & -1 & 9 \end{pmatrix}$$

2. (20pts) Consider the two-point boundary value problem

$$u'' + 2xu' - x^2u = x^2, \quad u(0) = 1, \quad u(1) = 0.$$

- (a) Let $h = 1/4$ and explicitly write out the difference equations, using *centered-difference quotients* for all derivatives (pages 352-353 of the textbook).
- (b) Convert the equations to a linear system $A\mathbf{x} = \mathbf{b}$, and then solve the system by hand or using Matlab.
3. (a) (15pts) Show that if a square matrix A has decomposition $A = LL^T$ with L nonsingular, then A is symmetric positive definite.
- (b) (5pts) Find the Cholesky factor L in the decomposition $A = LL^T$ for the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 9 \end{pmatrix}.$$

Show your work.

4. (10pts) Let A be a positive definite $n \times n$ matrix. For any $\mathbf{x} \in \mathbb{R}^n$ define $\|\mathbf{x}\|_A = (\mathbf{x}^T A \mathbf{x})^{1/2}$. Show that this defines a norm on \mathbb{R}^n .
5. (25pts) Given the data:

x	4.0	4.2	4.5	4.7	5.1	5.5	5.9	6.3	6.8	7.1
y	102.56	113.18	130.11	142.05	167.53	195.14	224.87	256.73	299.50	326.72

- (a) Construct the least squares polynomial of degree 1 and compute the error.
- (b) Construct the least squares polynomial of degree 3 and compute the error.
- (c) Construct the least squares approximation of the form $y = be^{ax}$ and compute the error. (*Hint: take logarithm on both sides.*)

Submit your code and results.