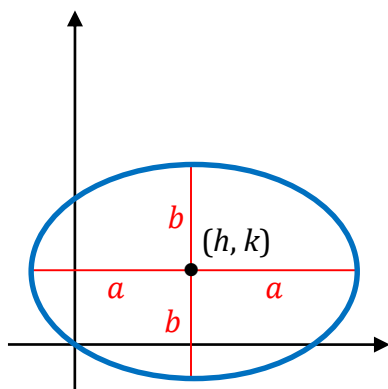# Summary of the Conic Sections

Definition: An ellipse is the set of all points in the plane the sum of whose distances from two fixed points (the foci) is constant.
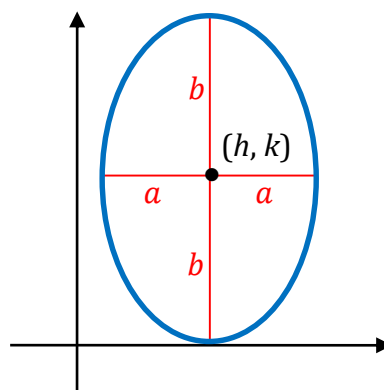
The standard equation of an ellipse is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Major axis is horizontal

$$a > b$$

$$c = \sqrt{a^2 - b^2}$$

Major axis is vertical

$$b > a$$

$$c = \sqrt{b^2 - a^2}$$

For both types of ellipses, the center is (h, k), and the vertices are the endpoints of the **major** axis.

Use the value of c to find the coordinates of each focus. The foci are always located on the major axis and are each c units away from the center.

Circles: If $a = b$, then the ellipse is a circle, and a and b will both be equal to the radius r. The foci of a circle are located at the same point—the center. Remember that the standard equation of a circle is
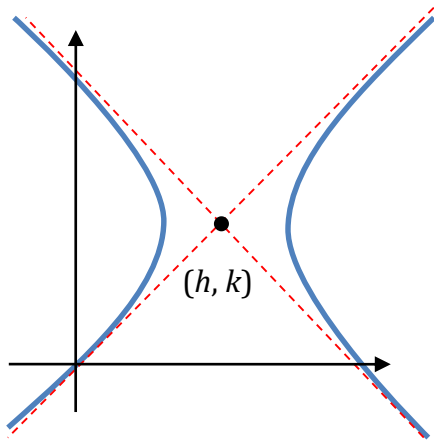
$$(x-h)^2 + (y-k)^2 = r^2$$
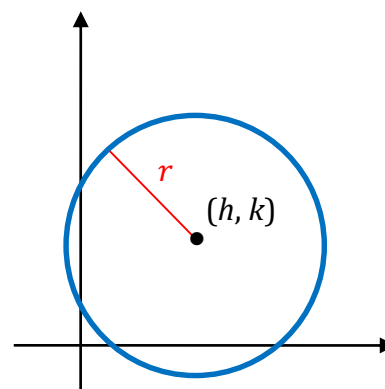
HYPERBOLAS

Definition: A hyperbola is the set of all points in the plane the difference of whose distances from two fixed points (the foci) is constant.

For hyperbolas that open left and right, the standard equation is
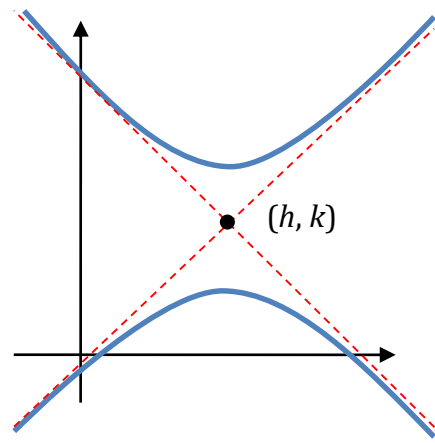
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

For hyperbolas that open up and down, the standard equation is

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Transverse axis = $2a$

Conjugate axis = $2b$

Foci are to the left and right of the center.

Transverse axis = $2b$

Conjugate axis = $2a$

Foci are above and below the center.

For both types of hyperbolas, the center is (h, k), and the vertices are the endpoints of the transverse axis (they are the turning points of the branches of the hyperbola).

Use the value $c = \sqrt{a^2 + b^2}$ to find the coordinates of each focus. The branches of a hyperbola will always bend towards the foci and away from the center.

Use the values of a and b to create a rectangular box around the center of the hyperbola. The diagonals of this box form the asymptotes. The equations of the asymptotes are
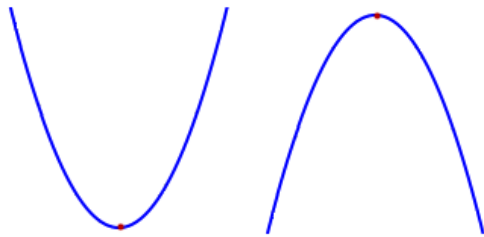
$$y - k = \pm \frac{b}{a}(x - h)$$

Definition: A parabola is the set of all points in the plane equidistant from a fixed line (the directrix) and a fixed point (the focus).

For a parabola that opens up or down, the standard equation is
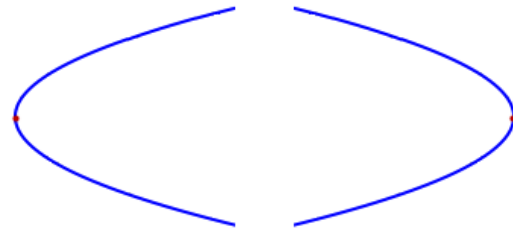
$$(x - h)^2 = 4p(y - k)$$

$p > 0$ $p < 0$

For a parabola that opens left or right, the standard equation is

$$(y - k)^2 = 4p(x - h)$$

$p > 0$ $p < 0$

For each parabola, the vertex is (h, k). The distance from the vertex to both the focus and directrix is given by $|p|$.

The graph of a parabola always bends towards its focus and away from its directrix (see the example below).

focus

vertex

directrix