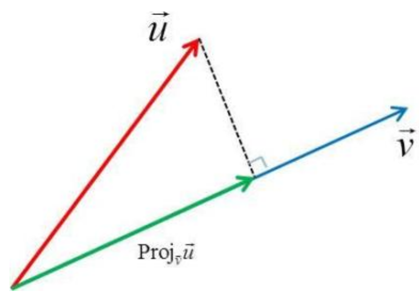
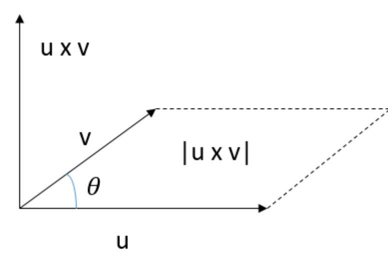
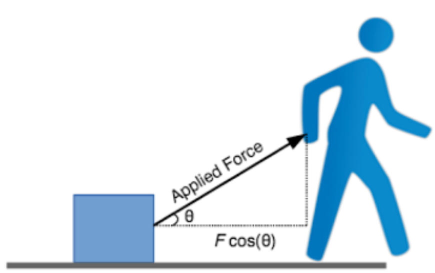
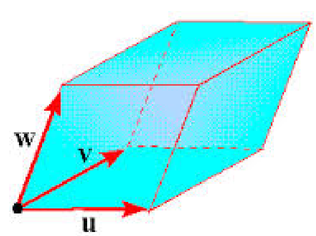
THE DOT AND CROSS PRODUCTS

Let $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be vectors and θ be the angle between $\mathbf{u}$ and $\mathbf{v}$.

<table>
<tr><th>Dot product: a scalar</th><th>Cross product: a vector</th></tr>
<tr><td>$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 = |\mathbf{u}||\mathbf{v}|\cos\theta$</td><td>$\mathbf{u} \times \mathbf{v} = (|\mathbf{u}||\mathbf{v}|\sin\theta)\mathbf{n}$
$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
$= (u_2v_3 - v_2u_3)\mathbf{i} - (u_1v_3 - v_1u_3)\mathbf{j} + (u_1v_2 - v_1u_2)\mathbf{k}$</td></tr>
<tr><td>$\mathbf{u} \cdot \mathbf{v} = 0$ if and only if $\mathbf{u} \perp \mathbf{v}$
$\mathbf{u} \cdot \mathbf{v} = \pm|\mathbf{u}||\mathbf{v}|$ if and only if $\mathbf{u} \parallel \mathbf{v}$</td><td>$\mathbf{u} \times \mathbf{v} = \mathbf{0}$ if and only if $\mathbf{u} \parallel \mathbf{v}$
(or $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$)</td></tr>
<tr><td>Angle between $\mathbf{u}$ and $\mathbf{v}$ is given by
$\theta = \cos^{-1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$</td><td>$\mathbf{u} \times \mathbf{v} \perp \mathbf{u}$ and $\mathbf{u} \times \mathbf{v} \perp \mathbf{v}$</td></tr>
<tr><td>Vector projection:
$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}\right)\left(\frac{\mathbf{v}}{|\mathbf{v}|}\right) = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$

</td><td>Area of parallelogram determined by $\mathbf{u}$ and $\mathbf{v}$:
Area = base $\cdot$ height $= |\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}|\sin\theta$, then area of triangle is $(1/2)|\mathbf{u} \times \mathbf{v}|$

</td></tr>
<tr><td>Work done by a force $\mathbf{F}$ in the direction of object displacement $\mathbf{D}$: $W = \mathbf{F} \cdot \mathbf{D} = |\mathbf{F}||\mathbf{D}|\cos\theta$

</td><td>Volume of parallelepiped determined by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$:
Volume $= |(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}| = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_2 \end{vmatrix}$

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