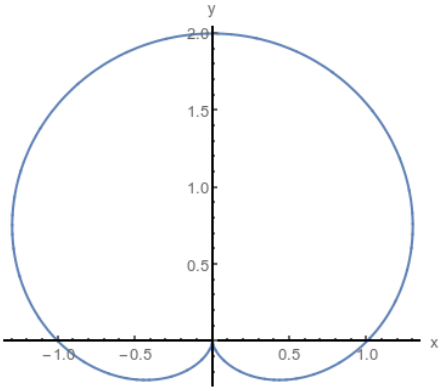
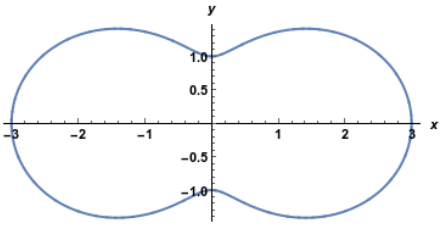
MTH 212 - MULTIVARIATE CALCULUS - PRACTICE PROBLEMS FOR EXAM I

Please be aware that this is NOT intended as a comprehensive list of all possible problem types!

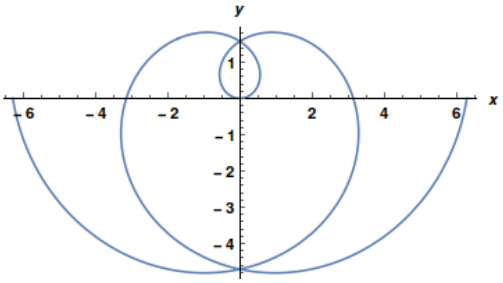
1. Consider the parametric curve given by $x = 4 + t^2$, $y = 5t^2 - t^3$, $1 \leq t \leq 5$.
   (a) Find the slope of the tangent line to the curve at the point where $t = 4$.
   (b) Find any t-values in the given interval $[1, 5]$ where the tangent line is horizontal.
   (c) Find $\dfrac{d^2y}{dx^2}$. Is the curve concave up or down for $1 < t < 5$?
2. Given the parametric equations $x = 4\cos t$ and $y = 2\sin t$, $0 \leq t \leq 2\pi$, idenify the curve by finding a Cartesian equation for it.
3. Identify the conic section represented by the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$.
4. Find parameterization for the line segment with endpoints $(-1, -3)$ and $(4, 1)$.
5. Replace the polar equations with equivalent Cartesian equations and idenify the graphs.
   (a) $r = 4\csc\theta$
   (b) $r^2 = 4r\cos\theta$
   (c) $r = \dfrac{4}{2\cos\theta - \sin\theta}$
   (d) $r = \csc\theta e^{r\cos\theta}$
6. Find the slopes of the cardioid $r = -1 + \sin\theta$ at the points $\theta = 0$ and $\theta = \pi$.



7. Find the area generated by revolving the curve $x = \cos t$, $y = 2 + \sin t$, $0 \leq t \leq 2\pi$, about the x-axis.
8. Find the area enclosed by the "peanut" $r = 2 + \cos(2\theta)$.



9. Set up, but DO NOT EVALUATE the integral to determine the length of the portion of the spiral $r = \theta$, for $-2\pi \leq \theta \leq 2\pi$.



10. Write equations or inequalities that correspond to the following descriptions of sets of points in space:

    (a) The plane going through the point $(3, -1, 1)$ parallel to the yz-plane.

    (b) The slab bounded by the planes $y = -1$ and $y = 0$ (planes included).

    (c) The sphere of radius 2 centered at the point $(1, 1, 1)$.

    (d) The half-space consisting of all the points on and below the xy-plane.

11. Find the center and the radius of the sphere $(x - \sqrt{2})^2 + (y + 0.5)^2 + (z + 3)^2 = 3$.

12. Find the distance between points $P_1(1, 1, 1)$ and $P_2(3, 3, 0)$.

13. Find a formula for the distance from the point $P(x, y, z)$ to the x-axis and z-axis.