MTH 212 - MULTIVARIATE CALCULUS - PRACTICE PROBLEMS FOR EXAM II

Please be aware that this is not intended as a comprehensive list of all possible problem types!

1. Consider the vector $\mathbf{u} = \overrightarrow{PQ}$ with initial point $P(4,3,0)$ and terminal point $Q(2,5,1)$.
   (a) Find the component form of $\mathbf{u}$.
   (b) Find the length of $\mathbf{u}$.
   (c) Express $\mathbf{u}$ as a product of its length and direction.
   (d) Find the unit vector in the direction of $\mathbf{u}$.
   (e) Find a vector of length 2 in the direction opposite to $\mathbf{u}$.

2. Find the angle between the vectors $\mathbf{u} = \langle 1,2,3\rangle$ and $\mathbf{v} = \langle 2,-3,-1\rangle$.

3. Given vectors $\mathbf{u} = \langle 3,2,1\rangle$, $\mathbf{v} = \langle 5,3,0\rangle$, and $\mathbf{w} = \langle -2,4,1\rangle$, calculate $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{w}$, and $\sqrt{2}\mathbf{u}$.

4. Let **P1** be a plane given by $2x+3y+z=5$ and **P2** be a plane given by $3x-2y=2$, and **L** is a line given by $\mathbf{r}(t) = \langle t, 1+t, 2-5t\rangle$. Circle all correct statements:
   (a) **P1** and **P2** are parallel
   (b) **P1** and **L** intersect
   (c) **P2** and **L** intersect
   (d) **L** is the intersection of **P1** and **P2**

5. Given vectors $\mathbf{u} = -2\mathbf{i}+4\mathbf{j}-4\mathbf{k}$ and $\mathbf{v} = -\mathbf{i}+\mathbf{j}+\mathbf{k}$, find:
   (a) the projection of $\mathbf{v}$ onto $\mathbf{u}$;
   (b) the cross product $\mathbf{u}\times\mathbf{v}$;
   (c) the area of the triangle with vertices $(0,0,0)$, $(-2,4,-4)$, and $(-1,1,1)$.

6. How much work does it take to slide a crate 20 m along a loading dock by pulling on it with 200-N force at an angle of $30°$ of the horizontal?

7. How do you determine whether two vectors are orthogonal, parallel, or neither?

8. Find the volume of the parallelepiped with the vertices $A(1,1,1)$, $B(2,0,3)$, $C(4,1,7)$, $D(3,-1,-2)$.

9. Determine the distance from point $(2,2,3)$ to the plane $x+2y+2z=4$.

10. Determine the distance from point $(1,2,-4)$ to the line $\mathbf{r}(t) = \langle 1+2t, 1-2t, -3+t\rangle$.

11. Find the point of intersection of line $\mathbf{r}(t) = \langle 1+t, 2-t, 3+2t\rangle$ with the plane $x+y+z=10$.

12. Find the vector parallel to the line of intersection of the planes $2x+z=5$ and $x+y-z=4$, then find parametric equations for this line (using this vector and any point of intersection of the planes).

13. Find the angle between the planes given by $x+y-z=6$ and $2x-2y+2z=2$. (Leave your answer as the arccosine of some number.)

14. Find the equation of the plane containing the lines $\mathbf{r}_1(t) = \langle -2 - t, 2t, 3 + 2t \rangle$ and $\mathbf{r}_2(s) = \langle 4 + s, 6 + 4s, -12 - 3s \rangle$. Write your answer in the form $Ax + By + Cz = D$.

15. Find the equation of the plane containing points $(1, 1, 0)$, $(0, 2, 1)$, and $(2, 1, 1)$. Write your answer in the form $Ax + By + Cz = D$.

16. Given the position of a particle in space at time t, $\mathbf{r}(t) = (2\ln(t + 1))\mathbf{i} + t^2\mathbf{j} + \frac{t^2}{2}\mathbf{k}$, compute the following:

    (a) the particle's velocity and acceleration at $t = 1$;

    (b) the particle's speed and direction of motion at $t = 1$.

17. Determine whether the lines $x = 1 + t$, $y = 2t$, $z = 1 + 3t$ and $x = 3s$, $y = 2s$, $z = 2 + s$ are parallel, skew, or intersecting. (If they are intersecting, find their point of intersection.)

18. Let $\mathbf{a}(t) = \langle 12t^2 + 2, 6t - \cos t, e^t \rangle$ represent acceleration and let $\mathbf{v}(0) = \langle 1, 0, -5 \rangle$ and $\mathbf{r}(0) = \langle 3, -3, 0 \rangle$. Find $\mathbf{v}(t)$ and $\mathbf{r}(t)$.

19. Determine the length of the curve $\mathbf{r}(t) = \langle 3t, \sin(4t), \cos(4t) \rangle$ from $t = 0$ to $t = 2\pi$.