MTH 212 - MULTIVARIATE CALCULUS - PRACTICE PROBLEMS FOR EXAM III

Please be aware that this is not intended as a comprehensive list of all possible problem types!

1. Given $\mathbf{r}(t) = \langle 5\sin t, 12, 5\cos t\rangle$, compute the unit tangent vector $\mathbf{T}(t)$.
2. Suppose $\mathbf{a} = \langle 2, 1, 1\rangle$ and $\mathbf{T} = \langle 3/5, 0, 4/5\rangle$ at some value of t. Compute a_T and a_N at t.
3. Find $\mathbf{T}$, $\mathbf{N}$, and κ for the space curve $\mathbf{r}(t) = (e^t\cos t)\mathbf{i} + (e^t\sin t)\mathbf{j} + 2\mathbf{k}$ and any time t and then at $t = 0$.
4. Find the **TNB**-frame for the space curve $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} - \mathbf{k}$ at $t = \pi/4$.
5. Evaluate $\displaystyle\lim_{(x,y)\to(1,0)} \frac{\sin y + \ln x}{x^2 + y^2}$.
6. Show that the function $f(x,y) = \dfrac{(x-1)^2 + y^2}{(x-1)^2 + 2y^2}$ has no limit as (x,y) approaches $(1,0)$.
7. Evaluate the limit or show that the limit does not exist. Show proper work.
   (a) $\displaystyle\lim_{(x,y)\to(0,\frac{\pi}{2})} \frac{\sin y\cos y}{ye^x}$.
   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4 + 3y^4}$.
8. Let $f(x,y,z) = x^2y + z^2 + 8$ and $P = (2,1,0)$. Find the unit vector which points in the direction of most rapid increase of $f(x,y,z)$ at P.
9. Suppose $\nabla f = \langle 3x^2 + 4y, \frac{5}{2}x + 16y\rangle$ for some function $f(x,y)$ and let P be the point $(2,0)$.
   (a) Compute the direction of greatest decrease at P (be sure it's a unit vector!)
   (b) Compute the directional derivative of f at P in the direction found in part (a).
   (c) Find all directions at which the directional derivative at P is 0.
10. Let $f(x,y) = \cos x + \sin y$. Give an equation for the plane tangent to the graph of $z = f(x,y)$ at $P(\frac{\pi}{2}, \pi, 0)$. Simplify your solution to the form $Ax + By + Cz = D$.
11. Consider the surface $F(x,y,z) = xy^2z + \ln(xy) - 1 = 0$. Find the equation of the tangent plane at the point $(e,1,0)$. Simplify your solution to the form $Ax + By + Cz = D$.
12. Consider the function $F(x,y,z) = \sin x + \cos y + e^z$.
    (a) Evaluate ∇F at the point $P(\pi, 0, \ln 2)$.
    (b) Find parametric equations for the normal line to the level surface $F(x,y,z) = 1$ at P.
13. Give the linearization of the function $f(x,y) = e^x + \sin y$ at the point $P = (0,\pi)$.
14. Give the linearization of the function $g(x,y) = x^2y + y$ at the point $P = (1,2)$.

15. (a) For a function $f(x, y, z)$ with $x = x(r, t)$, $y = y(r, t)$, and $z = z(r, t)$, what is the general multivariate chain rule for computing $\partial f/\partial r$?

    (b) Let $f(x, y, z) = x^2 + 2yz + 3z^2$, $x(r, t) = r^2 + \cos(\ln t)$, $y(r, t) = e^{t-1}$, and $z(r, t) = r^2 + t^2$. Evaluate $\partial f/\partial r$ at the point $(r, t) = (1, 1)$.

16. (a) For a function $f(r, s)$ with $r = r(x, y, z)$ and $s = s(x, y, z)$, what is the general multivariate chain rule for computing $\partial f/\partial y$?

    (b) Let $f(r, s) = rs + s^2$ with $r = xy^2$ and $s = \cos(yz)$. Evaluate $\partial f/\partial y$ at the point $(x, y, z) = (e, \pi/2, 4)$.

17. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $x^3z^2 - 5xy^5z = x^2 + y^3$

18. Find (but DO NOT classify) all critical points of $f(x, y) = x^2 - \frac{1}{18}(x - 1)y^2$.

19. The function $g(x, y) = x^3 + x^2 + 3y^2 - x - 12y + 11$ has critical points $(-1, 2)$ and $(\frac{1}{3}, 2)$. Classify these two points as saddle points and local extrema (max/min). Show proper work, computing any necessary values for classification.

20. Consider the function $f(x, y) = x^3 + 3x^2 + y^2 + 2y$. Fill in the four empty boxes in the following table. Each row corresponds to one of the two critical points. The columns provide the point, the value $f_{xx}f_{yy} - f_{xy}^2$ at the point, and the interpretation of that value (local min/max or saddle point).

Point	$f_{xx}f_{yy} - f_{xy}^2$	Min/Max/Saddle
$(0, -1)$		
	-12	

21. Consider the function $f(x, y) = x^3 + 8y^3 - 12xy$.

    (a) $(0, 0)$ is a critical point. Classify it as a min, a max, or a saddle point. Clearly identify the value of $f_{xx}f_{yy} - f_{xy}^2$ and f_{xx} to distinguish between a max and a min (if needed).

    (b) Find and classify all remaining critical points, clearly ientifying values of $f_{xx}f_{yy} - f_{xy}^2$ and f_{xx} when necessary.