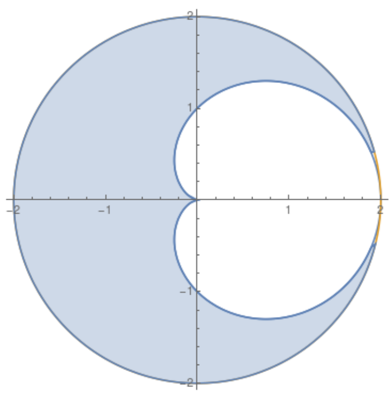
MTH 212 - MULTIVARIATE CALCULUS - PRACTICE PROBLEMS FOR EXAM IV

Please be aware that this is not intended as a comprehensive list of all possible problem types!

1. Find the volume of the solid bounded above by the surface $z = xye^{xy^2}$ and below by the rectangle R: $0 \leq x \leq 2$ and $0 \leq y \leq 1$.

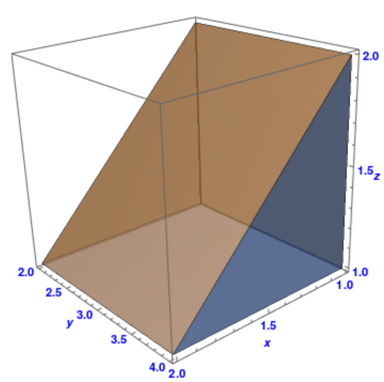
2. Set up but do not evaluate an integral using polar coordinates to find the area of the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, \sqrt{3})$.

3. Set up a double integral of the function $f(x, y) = x^2 + y^2$ over the region in the plane between the cardioid $r = 1 + \cos\theta$ and the circle of radius 2 centered at the origin, shown below. Use polar coordinates; simplify the integrand as much as you can, but do not evaluate the integral.



4. Find the average value of the function $g(x, y) = x \cos xy$ over the rectangle R: $0 \leq x \leq \pi$ and $0 \leq y \leq 1$.

5. Consider the triangular prism T shown below. The back wall lies on the plane $x = 1$, the side walls are triangles on the planes $y = 2$ and $y = 4$, the bottom lies on the plane $z = 1$, and the other face (the rectangle face in the front) lies on the plane $x + z = 3$.



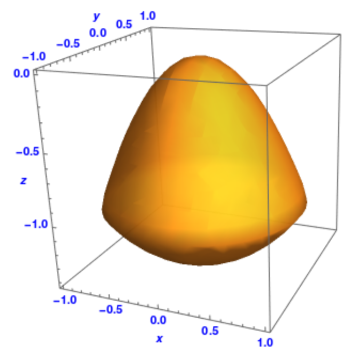
   (a) Set up the triple integral to find the volume of T using the variable order $dzdydx$.
   (b) Set up the triple integral to find the volume of T using the variable order $dydxdz$.

(c) Find the volume of T using either (a) or (b) or any other method.

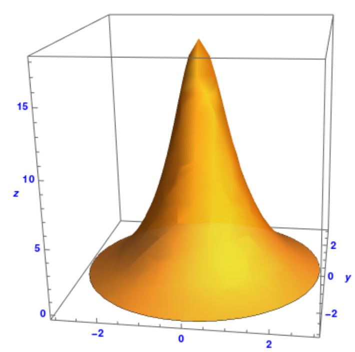
6. Let S be the solid above the sphere $x^2+y^2+z^2=2$ and below the paraboloid $z+x^2+y^2=0$. Notice that two surfaces intersect in the circle $x^2+y^2=1$ on the plane $z=-1$.

   (a) Set up (but do not evaluate) one triple integral to find the volume of S using *cylindrical* coordinates.

   (b) Set up (but do not evaluate) two triple integrals (added together) to find the volume of S using *spherical* coordinates.



7. Find the volume of the solid bounded by $z=0$ and $z=\dfrac{20}{1+x^2+y^2}-2$, shown below.



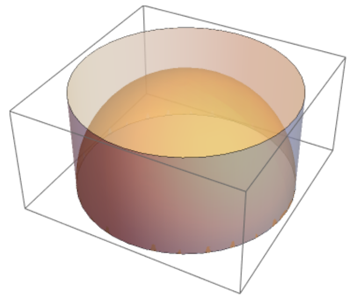
8. Consider the region D inside the cylinder $x^2+y^2=1$, outside the sphere $x^2+y^2+z^2=1$, and with z between 0 and 1. A view from above is shown below.

   (a) Set up (but do not evaluate) one (or more) triple integral(s) to find the volume of D using *cylindrical* coordinates.

   (b) Set up (but do not evaluate) one (or more) triple integral(s) to find the volume of D using *spherical* coordinates.

9. Consider the region R within the cylinder $x^2+y^2\le 4$, bounded below by $z=0$ and above by $z=2-y$. Assume a mass density is $\delta=z$.

   (a) Evaluate the integral representing the mass of the solid, using cylindrical coordinates.



(b) Set up but do not evaluate the integral representing M_{yz}, again with cylindrical coordinates.

10. Consider the thin plate T consisting of the portion of unit disk in the first quadrant of the xy-plane. Let $\delta(x, y) = \sqrt{x^2 + y^2}$ be the density of T. Set up but do not evaluate an integral to find the mass of T using polar coordinates.

11. Set up the line integral of $f(x, y, z) = x + y + z$ over the straight line segment from point $(1, 2, 3)$ to point $(0, -1, 1)$.

12. Evaluate $\int_C (xy + y + z)\, ds$ where C is the curve $\mathbf{r}(t) = 2t\mathbf{i} + t\mathbf{j} + (2 - 2t)\mathbf{k}$, $0 \le t \le 1$.