MTH 212 - MULTIVARIATE CALCULUS - PRACTICE PROBLEMS FOR FINAL

Please be aware that this is not intended as a comprehensive list of all possible problem types. ONLY EXAMPLES OF PROBLEMS FROM THE POST-EXAM IV MATERIAL ARE INCLUDED HERE!

1. Find the work done by the field $\mathbf{F} = xy\mathbf{i} + y\mathbf{j} - yz\mathbf{k}$ over the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$ in the direction of increasing t.

2. Find the flux of the field $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j}$ across the closed semicircular path that consists of the arch $\mathbf{r}_1(t) = \cos t\mathbf{i} + \sin t\mathbf{j}$, $0 \le t \le \pi$, followed by the line segment $\mathbf{r}_2(t) = t\mathbf{i}$, $-1 \le t \le 1$.

3. Is the vector field $\langle yz - \sin x \sin z, xz + \cos x \sin z, xy + \cos x \cos z\rangle$ conservative? Justify your answer.

4. $\mathbf{F} = \langle 2x, -3z, -3y + 3z^2\rangle$ is a conservative vector field with potential function $f = x^2 - 3yz + z^3$. Find the work done when moving along a straight line segment from $(1, 2, 0)$ to $(1, 0, 1)$ through F.

5. $\mathbf{G} = \langle e^x + y\cos(xy), 2yz + x\cos(xy), y^2\rangle$ is a conservative vector field. Find the potential function g so that $g(0, 0, 0) = 0$.

6. Set up (but do not evaluate) an integral for the surface area of the paraboloid $y = x^2 + z^2$, sliced by the planes $y = 1$ and $y = 16$ (you can use parameterization $\mathbf{r}(r, \theta) = \langle r\cos\theta, r^2, r\sin\theta\rangle$).

7. Let S be the portion of the paraboloid $z = 1 - x^2 - y^2$ above the plane $z = 0$ and C, the unit circle in the plane $z = 0$ (with counterclockwise orientation), be the boundary of S. According to Stokes' Theorem, circulation around C of vector field $\mathbf{F} = \langle xy, -y^2, 0\rangle$ can be computed as a line integral or as a surface integral.

   (a) Circle the letter of the option below that correctly computes the circulation around C as a line integral:

      i. $\int_0^{2\pi} 1\ dt$
      ii. $\int_0^{2\pi} \cos t \sin t\ dt$
      iii. $\int_0^{2\pi} (-\sin^2 t - 2\sin t\cos t)\ dt$
      iv. $\int_0^{2\pi} -2\cos t \sin^2 t\ dt$
      v. $\int_0^{2\pi} (\cos t - \sin t)\ dt$
      vi. none of the above

   (b) Circle the letter of the option below that correctly computes the circulation around C as a surface integral over S (with outward pointing normal):

      i. $\int_0^{2\pi}\int_0^1 -r\cos\theta\ drd\theta$
      ii. $\int_0^{2\pi}\int_0^1 -r^2\cos\theta\ drd\theta$
      iii. $\int_0^{2\pi}\int_0^1 r^2\sin\theta\ drd\theta$
      iv. $\int_0^{2\pi}\int_0^1 r^3\cos\theta\ drd\theta$
      v. $\int_0^{2\pi}\int_0^1 r\ drd\theta$
      vi. none of the above

8. Consider the portion of the paraboloid $z = 4 - x^2 - y^2$ above the xy-plane. One parameterization for this surface is $\mathbf{r}(r, \theta) = \langle r\cos\theta, r\sin\theta, 4 - r^2\rangle$ with $0 \le r \le 2$ and $0 \le \theta \le 2\pi$. Set up (but do not evaluate) a surface integral of $g(x, y, z) = z^2$ over this surface.

9. Consider the field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + xy\mathbf{k}$. Let S be the surface defined by $z = xy$ with $0 \le x \le 1$ and $0 \le y \le 2$. Write down a parameterization for S and set up an integral that computes the flux of $\mathbf{F}$ across S, in the direction of the normal vector $\mathbf{n}$.

10. Use either version of Green's Theorem to evaluate $\oint_C y^3 dx - x^3 dy$ as a double integral, where C is the circle of radius 3 centered at the origin and with counterclockwise orientation.

11. Using Green's Theorem and the appropriate choice of variables, evaluate $\oint_C y^2 dx + 3xy dy$ where C is the boundary of the region in the upper half plane bounded by $x^2 + y^2 = 1$, $x^2 + y^2 = 4$, and $y = 0$.

12. Let S be a surface of the cylinder given by $x^2 + y^2 = 4$ with $0 \le z \le 3$, including the top and the bottom. Set up a *triple* integral in cylindrical coordinates to compute the flux of $\mathbf{F} = \langle xy, yz, xz\rangle$ across S (but do not evaluate it).