Formula Sheet

- The derivative of the function f in the direction of a unit vector $\mathbf{u}$ at the point P_0 is $D_{\mathbf{u}}f(P_0) = \nabla f(P_0) \cdot \mathbf{u}$.
- The linearization of $f(x, y)$ at a point $P_0 = (x_0, y_0)$ is $L(x, y) = f(P_0) + f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0)$.
- *The Second Derivative Test for Functions of Two Variables:*
  Let $f(x, y)$ be a twice differentiable function, and assume its second partial derivatives are continuous. Let (a, b) be a critical point for f, and define the Hessian of f at (a, b) to be
  $H(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$.
  - If $H(a, b) < 0$, then (a, b) is a saddle point.
  - If $H(a, b) > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a local min.
  - If $H(a, b) > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a local max.
  - If $H(a, b) = 0$, the test is inconclusive.
- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos\theta$ and $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin\theta$
- The arc length of a curve $\mathbf{r}(t)$ for $a \leq t \leq b$ is given by

  $$L = \int_a^b |\mathbf{v}(t)|dt,$$

  where $\mathbf{v}(t) = \frac{d\mathbf{r}}{dt}$.
- Distance from point S to a line passing through point P, parallel to $\mathbf{v}$: $d = \frac{|\mathbf{PS} \times \mathbf{v}|}{|\mathbf{v}|}$.
- Given point S in space, and a plane with normal $\mathbf{n}$ and point P on the plane, the distance from S to the plane is $d = \frac{|\mathbf{PS} \cdot \mathbf{n}|}{|\mathbf{n}|}$.
- $\mathbf{N} = (d\mathbf{T}/dt)/|d\mathbf{T}/dt|$ (principal unit normal vector)
- $\kappa = (1/|\mathbf{v}|)|d\mathbf{T}/dt|$ (curve curvature).
- $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$, where $a_T = \frac{d(|\mathbf{v}|)}{dt}$ and $a_N = \sqrt{|\mathbf{a}|^2 - a_T^2}$.
- Trigonometric identities:

  $$\sin^2 x + \cos^2 x = 1$$

  $$\tan^2 x + 1 = \sec^2 x$$

  $$\sin 2x = 2 \sin x \cos x$$

  $$\cos 2x = \cos^2 x - \sin^2 x$$

  $$\sin^2 x = (1 - \cos 2x)/2$$

  $$\cos^2 x = (1 + \cos 2x)/2$$

- For a plane curve given by $x = f(t)$ and $y = g(t)$, the derivative is given by

  $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

  provided $dx/dt \neq 0$.

- Areas of surfaces of revolution:

  about x-axis ($y \geq 0$): $S = \int_a^b 2\pi y \sqrt{[dx/dt]^2 + [dy/dt]^2}\ dt$
  about y-axis ($x \geq 0$): $S = \int_a^b 2\pi x \sqrt{[dx/dt]^2 + [dy/dt]^2}\ dt$

- Polar coordinates:

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ r &= \sqrt{x^2 + y^2} \\ \tan\theta &= \frac{y}{x} \end{aligned}$$

- The area enclosed by a polar curve $r = f(\theta)$ between two angles θ_1 and θ_2 is given by

$$\frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 d\theta = \frac{1}{2}\int_{\theta_1}^{\theta_2} (f(\theta))^2\, d\theta.$$