

Formula Sheet

- *Polar Coordinates:*

- $x = r \cos \theta, \quad y = r \sin \theta, \quad r = \sqrt{x^2 + y^2}, \quad \tan \theta = \frac{y}{x}$
 - $dA = r \, dr d\theta$

- *Cylindrical Coordinates:*

- $x = r \cos \theta, \quad y = r \sin \theta, \quad z = z, \quad r = \sqrt{x^2 + y^2}, \quad \tan \theta = \frac{y}{x}$
 - $dV = dz \, r \, dr d\theta$

- *Spherical Coordinates:*

- $x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi$
 - $\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}, \quad r = \rho \sin \phi$
 - $dV = \rho^2 \sin \phi \, d\rho d\phi d\theta$

- The center of mass $(\bar{x}, \bar{y}, \bar{z})$ of a three-dimensional object D with density $\delta(x, y, z)$: $\bar{x} = \frac{\int \int \int_D x \delta dV}{\int \int \int_D \delta dV}, \bar{y} = \frac{\int \int \int_D y \delta dV}{\int \int \int_D \delta dV}, \bar{z} = \frac{\int \int \int_D z \delta dV}{\int \int \int_D \delta dV}$. ($\int \int \int_D \delta dV$ is the mass of the object.)

- The derivative of the function f in the direction of a unit vector \mathbf{u} at the point P_0 is $D_{\mathbf{u}}f(P_0) = \nabla f(P_0) \cdot \mathbf{u}$.

- The linearization of $f(x, y)$ at a point $P_0 = (x_0, y_0)$ is $L(x, y) = f(P_0) + f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0)$.

- *The Second Derivative Test for Functions of Two Variables:*

Let $f(x, y)$ be a twice differentiable function, and assume its second partial derivatives are continuous. Let (a, b) be a critical point for f , and define the Hessian of f at (a, b) to be

$$H(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2.$$

- If $H(a, b) < 0$, then (a, b) is a saddle point.
 - If $H(a, b) > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a local min.
 - If $H(a, b) > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a local max.
 - If $H(a, b) = 0$, the test is inconclusive.

- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos \theta$ and $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin \theta$

- The arc length of a curve $\mathbf{r}(t)$ for $a \leq t \leq b$ is given by

$$L = \int_a^b |\mathbf{v}(t)| dt,$$

where $\mathbf{v}(t) = \frac{d\mathbf{r}}{dt}$.

- Distance from point S to a line passing through point P , parallel to \mathbf{v} : $d = \frac{|\mathbf{PS} \times \mathbf{v}|}{|\mathbf{v}|}$.
- Given point S in space, and a plane with normal \mathbf{n} and point P on the plane, the distance from S to the plane is $d = \frac{|\mathbf{PS} \cdot \mathbf{n}|}{|\mathbf{n}|}$.
- Principal unit normal vector $\mathbf{N} = (d\mathbf{T}/dt)/|d\mathbf{T}/dt|$
- Curve curvature $\kappa = (1/|\mathbf{v}|)|d\mathbf{T}/dt|$.

- Acceleration $\mathbf{a} = a_T \mathbf{T} + a_N \mathbf{N}$, where $a_T = \frac{d(|\mathbf{v}|)}{dt}$ and $a_N = \sqrt{|\mathbf{a}|^2 - a_T^2}$.
- For a plane curve given by $x = f(t)$ and $y = g(t)$, the derivative is given by

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

provided $dx/dt \neq 0$.

- Areas of surfaces of revolution:

about x -axis ($y \geq 0$): $S = \int_a^b 2\pi y \sqrt{[dx/dt]^2 + [dy/dt]^2} dt$

about y -axis ($x \geq 0$): $S = \int_a^b 2\pi x \sqrt{[dx/dt]^2 + [dy/dt]^2} dt$

- The area enclosed by a polar curve $r = f(\theta)$ between two angles θ_1 and θ_2 is given by

$$\frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta = \frac{1}{2} \int_{\theta_1}^{\theta_2} (f(\theta))^2 d\theta.$$

- Trigonometric identities:

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin^2 x = (1 - \cos 2x)/2$$

$$\cos^2 x = (1 + \cos 2x)/2$$