Formula Sheet

- For a differential equation of the form
$$\frac{dx}{dt} + p(t)x(t) = q(t),$$
the general solution is in the form
$$x(t) = \frac{1}{\mu(t)}\Big(\int \mu(t)q(t)\ dt + C\Big),$$
where
$$\mu(t) = e^{\int p(t)dt}.$$

- For a differential equation of the form
$$\frac{dx}{dt} = p(t)x + q(t)x^n,\ \ n \neq 0, 1,$$
use the substitution $v(t) = (x(t))^{1-n}$.

- Newton's Law of Cooling: $T'(t) = k(A - T(t))$.

- Logistic growth equation: $P'(t) = rP(t)(1 - P(t)/N)$.

- A differential equation of the form
$$M(x,y)dx + N(x,y)dy = 0$$
is exact if and only if $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$. The general solution of an exact equation is of the form $F(x,y) = C$ where $\frac{\partial F}{\partial x} = M$ and $\frac{\partial F}{\partial y} = N$.

- For an initial value problem $x'(t) = f(t,x)$, $x(t_0) = x_0$, Euler's Method finds approximation of solution by
$$x_{i+1} = x_i + f(t_i, x_i)\Delta t$$
$$t_{i+1} = t_i + \Delta t$$
$(i = 1, 2, 3, \ldots)$, given a stepsize Δt.

- Errors in numerical methods:

Method	*Error*
Euler's	$\sim \Delta t$
Improved Euler's	$\sim (\Delta t)^2$
4th-Order Runge-Kutta	$\sim (\Delta t)^4$

- The Wronskian of two solutions $x_1(t)$ and $x_2(t)$ of the second-order linear homogeneous equation $x'' + p(t)x' + q(t)x = 0$ is the determinant
$$W(x_1, x_2)(t) = \begin{vmatrix} x_1(t) & x_2(t) \\ x_1'(t) & x_2'(t) \end{vmatrix}.$$
If $W(x_1, x_2)(t) \neq 0$ for all values of t, then $x = C_1x_1 + C_2x_2$ is a general solution of $x'' + p(t)x' + q(t)x = 0$.