

Homework 5 due April 12th Numerical Analysis, Spring 2023

Please show your work and hand in your MATLAB codes and outputs. (You can use the MATLAB function *diary* to copy your command window output.)

1. Let $f(x) = \sin x$ and use the forward difference formula $f'(x) \approx \frac{f(x+h)-f(x)}{h}$ to approximate $f'(\pi/6) = \sqrt{3}/2$. In MATLAB, produce a table with columns h , $\frac{\sin(\pi/6+h)-\sin(\pi/6)}{h}$, and the error of approximation $\left| \frac{\sin(\pi/6+h)-\sin(\pi/6)}{h} - \sqrt{3}/2 \right|$, for $h = 10^{-k}$, $k = 1, 2, \dots, 16$. You can use *fprintf* with appropriate options to display 15 places in your error values. Draw a log-log scale plot of the error (log error vs log h). When is the best accuracy achieved?
2. Let $f(x) = \sin x$ and use the centered difference formula $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$ to approximate $f'(\pi/6) = \sqrt{3}/2$. In MATLAB, produce a table with columns h , $\frac{\sin(\pi/6+h)-\sin(\pi/6-h)}{2h}$, and the error of approximation, $\left| \frac{\sin(\pi/6+h)-\sin(\pi/6-h)}{2h} - \sqrt{3}/2 \right|$, for $h = 10^{-k}$, $k = 1, 2, \dots, 16$. Draw a log-log scale plot of the error (log error vs log h). When is the best accuracy achieved?
3. Use the same function f from Exercises 1 and 2 to approximate $f''(\pi/6) = -\sin(\pi/6) = -1/2$ using $f''(x) \approx \frac{f(x+h)-2f(x)+f(x-h)}{h^2}$ and the same range for h . Draw a log-log scale plot of the error vs h . Discuss the results. When do we expect to see the smallest error?
4. Using *chebfun*.
 - (a) Define the Runge function in *chebfun* by typing

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f = chebfun('1./(1+x.^2)', [-5,5])
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Next, differentiate f by typing

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fp = diff(f)
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fpp = diff(f,2) % we can differentiate up to any order k: diff(f,k)
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Check the lengths of f , fp , and fpp to see the degrees of the polynomials representing f , fp and fpp .
Plot f , fp , and fpp in one figure, using *legend* to annotate the graph.
 - (b) Use *chebfun* to evaluate $f''(x)$ for $f(x) = \sin x$ and $x = \pi/6$. What is the degree of the interpolation polynomial that it produces for f and f'' , and what is the error in its approximation to $f''(\pi/6)$?
5. Chapter 9, Exercises 2 and 5.
6. *MTH 464* students also do Exercise 7 from Chapter 9.
(This can be a bonus problem for the other students.)