Homework 5 due April 3rd
Numerical Analysis, Spring 2019

Please show your work. Whenever needed, also hand in your MATLAB codes and outputs. (You can use the MATLAB function *diary* to copy your command window output.)

1. Let $f(x) = \sin x$ and use the forward difference formula $f'(x) \approx (f(x+h) - f(x))/h$ to approximate $f'(\pi/6) = \sqrt{3}/2$. In MATLAB, produce a table with columns h, $(\sin(\pi/6 + h) - \sin(\pi/6))/h$, and the error of approximation, $|(\sin(\pi/6+h) - \sin(\pi/6))/h - \sqrt{3}/2|$, for $h = 10^{-k}$, $k = 1, 2, \ldots, 16$. You can use *fprintf* with appropriate options to display 15 places in your error values. Draw a log-log scale plot of the error (log error vs log h). (Recall: the best accuracy is achieved when $h \approx \sqrt{\epsilon_{machine}} \approx 10^{-8}$.)

2. Let $f(x) = \sin x$ and use the centered difference formula $f'(x) \approx (f(x+h) - f(x-h))/(2h)$ to approximate $f'(\pi/6) = \sqrt{3}/2$. In MATLAB, produce a table with columns h, $(\sin(\pi/6+h) - \sin(\pi/6-h))/(2h)$, and the error of approximation, $|(\sin(\pi/6+h) - \sin(\pi/6-h))/(2h) - \sqrt{3}/2|$, for $h = 10^{-k}$, $k = 1, 2, \ldots, 16$. Draw a log-log scale plot of the error (log error vs log h). (Recall: the best accuracy is achieved when $h \approx \sqrt[3]{\epsilon_{machine}} \approx 10^{-5}$.)

3. Use the same function as in Exercises 1 and 2 above and approximate $f''(\pi/6) = -\sin(\pi/6) = -1/2$ using $f''(x) \approx (f(x+h) - 2f(x) + f(x-h))/(h^2)$ and the same range for h. Draw a log-log scale plot of the error vs h. Discuss the results. (We expect to see that the smallest error will occur when $h \approx \sqrt[4]{\epsilon_{machine}} \approx 10^{-4}$.)

4. Using *chebfun*.

   (a) Define the Runge function in *chebfun* by typing

   ```
   f = chebfun('1./(1+x.^2)',[-5,5])
   ```

   Next, differentiate f by typing:

   ```
   fp = diff(f)
   fpp = diff(f,2) % we can differentiate up to any order k: diff(f,k)
   ```

   Check the lengths of f, fp, and fpp to see the degrees of the polynomials representing f, fp and fpp.

   Plot f, fp, and fpp in one figure, using *legend* to annotate the graph.

   (b) Use *chebfun* to evaluate $f''(x)$ for $f(x) = \sin x$ and $x = \pi/6$. What is the degree of the interpolation polynomial that it produces for f and f'', and what is the error in its approximation to $f''(\pi/6)$?

5. Chapter 9, Exercise 2.
6. *MTH 464* student also does Exercise 5 from Chapter 9. This can be a bonus problem for the other students.