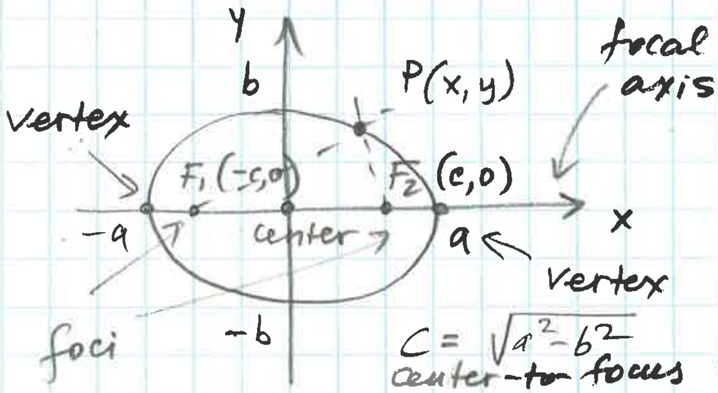
A.4 Conic Sections

Parabolas, ellipses, hyperbolas → called conics because they are formed by cutting a double cone w/ a plane

(Goal : express the conics in polar coord's in § 10.6.)

Ellipse: a set of points in a plane whose distances from two $\underbrace{\text{fixed points}}_{\text{foci}}$ have a constant sum.



General equation

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

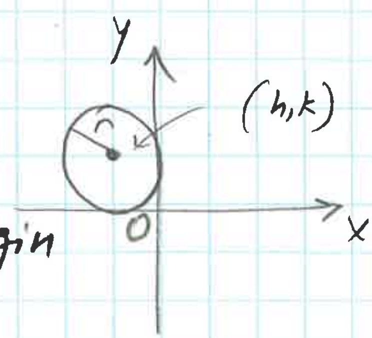
(h, k) - center

If $(h,k) = (0,0) \implies \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

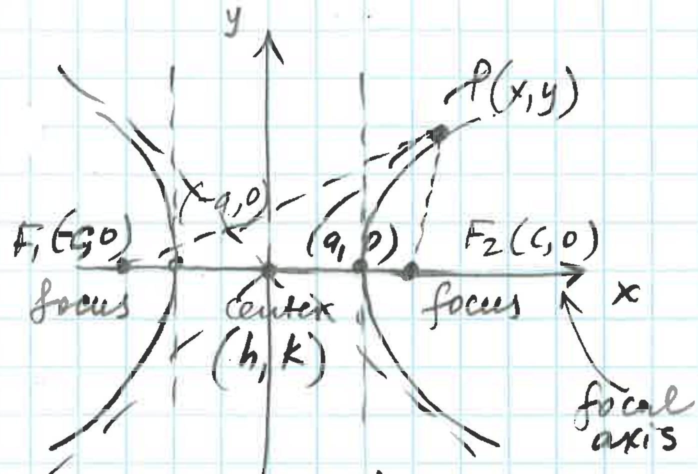
Circle: a particular case of an ellipse, when $a = b$

$$(x-h)^2 + (y-k)^2 = r^2$$

$x^2 + y^2 = r^2$ - centered at the origin



Hyperbola: a set of points in a plane whose distances from two $\underbrace{\text{fixed points}}_{\text{foci}}$ have a constant difference.



oblique asymptotes

$$y-k = \pm \frac{b}{a}(x-h)$$

Vertices $(\pm a, 0)$

foci $(\pm c, 0)$

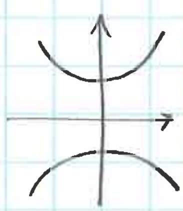
center (h,k)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$(b^2 = c^2 - a^2)$

$$(h,k) = (0,0) \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

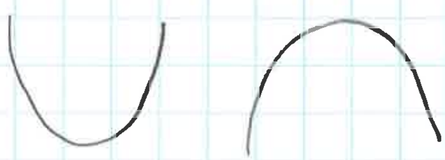
If foci on the y-axis: $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$



Center-to-focus distance: $c = \sqrt{a^2+b^2}$

Parabola: a set of all point in a plane equidistant from a fixed point (focus) and a fixed line (directrix).

a)



$p > 0$ $p < 0$

opens up/down vertex

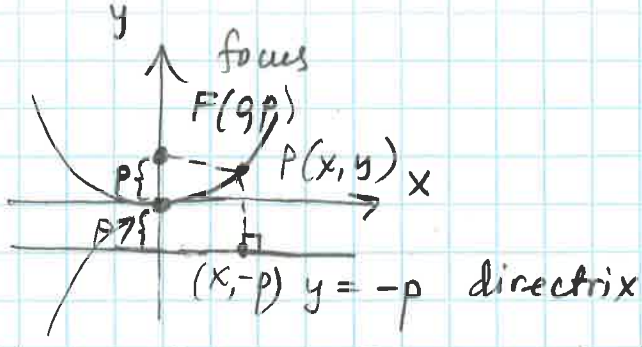
$$(x-h)^2 = 4p(y-k)$$

if $(h,k) = (0,0) \Rightarrow$

$x^2 = 4py$ or

$$y = \frac{x^2}{4p}$$

Ex:



p - focal length

Vertex lies half way between directrix and focus

b)

$p > 0$ $\quad$ $p < 0$

opens right / left

$$(y-k)^2 = 4p(x-h)$$

or, if $(h,k) = (0,0)$

$$\Rightarrow \quad y^2 = 4px \quad \text{or}$$

$$x = \frac{y^2}{4p}$$