A.4 Conic Sections

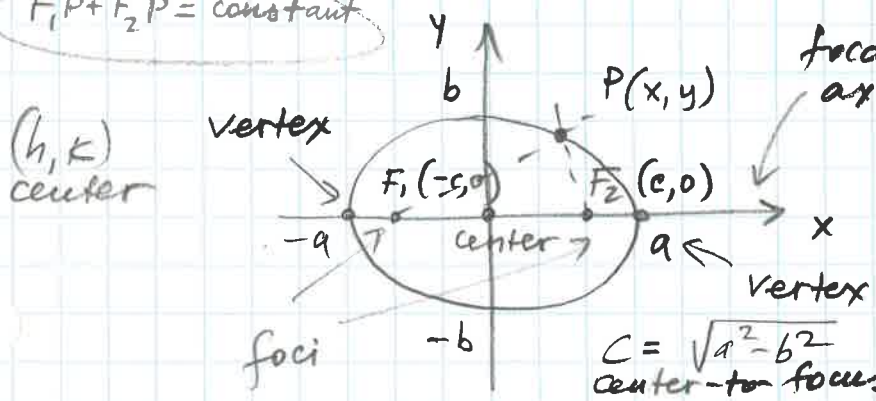
Parabolas, ellipses, hyperbolas → called conics because they are formed by cutting a double cone w/ a plane

(Goal: express the conics in polar coord's in §10.6.)

Ellipse: a set of points in a plane whose distances from two fixed points have a constant sum.

foci

$F_1P + F_2P = \text{constant}$



General equation

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

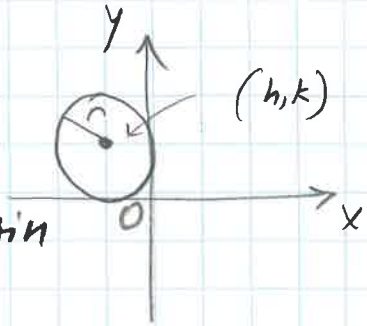
$c = \sqrt{a^2 - b^2}$ or $c = \sqrt{b^2 - a^2}$ is center-to-focus distance

If $(h,k) = (0,0) \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Circle: a particular case of an ellipse, when $a = b$

$$(x-h)^2 + (y-k)^2 = r^2$$

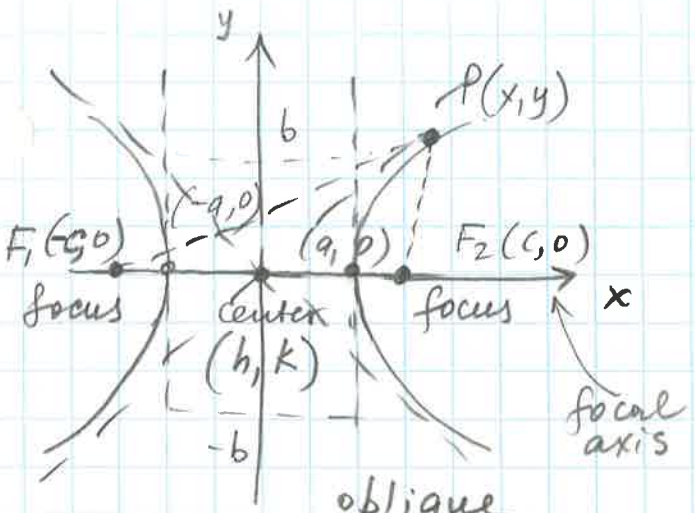
$x^2 + y^2 = r^2$ – centered at the origin



Hyperbola: a set of points in a plane whose distances from two fixed points have a constant difference.

foci

$F_1P - F_2P = \text{constant}$



Vertices $(\pm a, 0)$

foci $(\pm c, 0)$

center (h,k)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

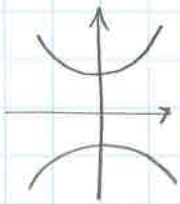
$(b^2 = c^2 - a^2)$

oblique asymptotes

$$y - k = \pm \frac{b}{a}(x-h)$$

$(h,k) = (0,0) \Rightarrow \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

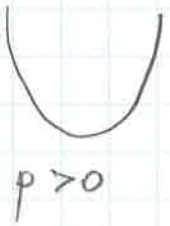
If foci on the y-axis: $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$



Center-to-focus distance: $c = \sqrt{a^2 + b^2}$

<u>Parabola</u>: a set of all point in a plane equidistant from a fixed point (<u>focus</u>) and a fixed line (<u>directrix</u>).

a)



$(x-h)^2 = 4p(y-k)$

if $(h,k) = (0,0) \Rightarrow$

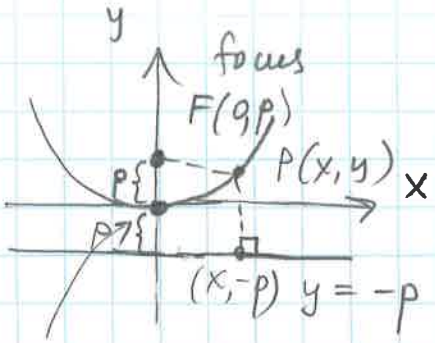
$x^2 = 4py$ or

$y = \frac{x^2}{4p}$

opens up / down

vertex

Ex:



p - focal length.

Vertex lies half way between directrix and focus

b)

$p > 0$ $p < 0$

opens right / left

$(y-k)^2 = 4p(x-h)$

or, if $(h,k) = (0,0)$

$\Rightarrow$ $y^2 = 4px$ or

$x = \frac{y^2}{4p}$