2 Parametric Curves:

intersection pts vs collision pts

Intersection pts are points which are on both sets of parametric equations (i.e., where the curves cross each other).

Collision pts are intersection pts that occur at the same value of the parameter.

Example: Consider 2 curves:

(1) $\begin{cases} x = \sin t \\ y = -\cos t \end{cases}$ $-2\pi \le t \le 2\pi$ (circle)

(2) $\begin{cases} x = s \\ y = -s \end{cases}$ (a line $y = -x$)

note: I use different parameters (t and s)!

Intersections pts are where

$\sin t = s$ and $-\cos t = -s$

$\Rightarrow \sin t = \cos t \Rightarrow \tan t = 1 \Rightarrow$

$t = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$ $\left(-2\pi \le t \le 2\pi\right)$

$\Rightarrow$ intersection pts are (plug t!) $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

Are some of them collision pts? When

$t = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$, $s = \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}$,

respectively. So, since $t \neq s \Rightarrow$ no collision pts!

Another example:

$$\begin{cases} x = t \\ y = t^2 \end{cases} \quad t > 0 \qquad \text{and} \qquad \begin{cases} x = \frac{1}{2}s + 1 \\ y = 2s \end{cases} \quad s > 0$$

DIY: Check that intersection occurs at $(x,y) = (2,4)$ ($t = 2$)

("do it yourself") Is it a collision pt?

$t = 2 \Rightarrow x = 2 \Rightarrow 2 = \frac{1}{2}s + 1 \Rightarrow s = 2$! Yes.

OR: check directly by setting

$$\begin{aligned} t &= \tfrac{1}{2}t + 1 \\ t^2 &= 2t \end{aligned} \quad \Rightarrow \quad t = 2 \Rightarrow (x,y) = (2,4)$$

(use the same parameter!)

So, $(2,4)$ is a collision pt (when $t = 2$)