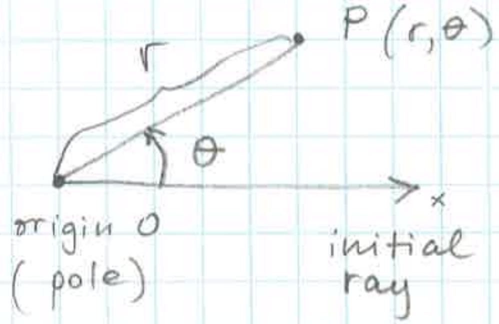
§10.3 Polar Coordinates.



→ polar coordinate pair;

r - directed distance from O to P,

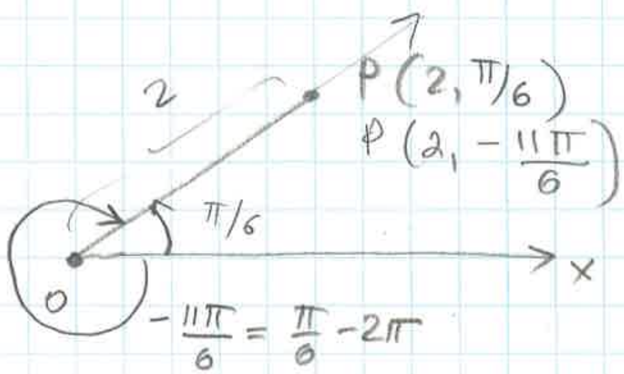
θ - directed angle from the initial ray to OP.

$\theta > 0$ if measured counterclockwise
$\theta < 0$ if measured clockwise
} as in trigonometry

$P(x,y)$ is defined uniquely in Cartesian coordinates x & y, while

$P(r,\theta)$ has infinitely many pairs in polar coordinates, e.g. $P(2,\frac{\pi}{6}) = P(2, \frac{-11\pi}{6})$

<u>Example 1</u>. Find all names for $P(2,\pi/6)$ (pairs)



1) $\underline{r=2}$, $\theta = \frac{\pi}{6}$ (goes counterclockwise)

$\Rightarrow \pi/6, \frac{\pi}{6}+2\pi, \frac{\pi}{6}-2\pi, \frac{\pi}{6}+4\pi, \frac{\pi}{6}-4\pi, \dots$

(Note $\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6} < 0$!)

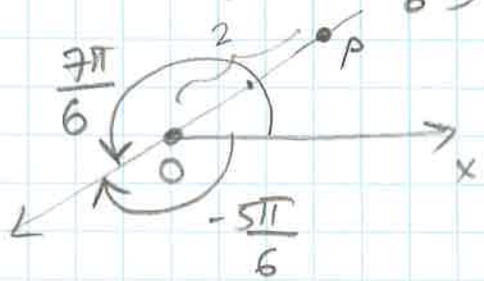
2) we can allow $r=-2$! θ

Angles are: $-\frac{5\pi}{6}, -\frac{5\pi}{6} \pm 2\pi, -\frac{5\pi}{6} \pm 4\pi, \dots$

For instance,

1) go $\frac{5\pi}{6}$ clockwise and measure <u>backward</u> 2 units:

$$P\left(-2, -\frac{5\pi}{6}\right) = P\left(-2, \frac{7\pi}{6}\right)$$



2) or go $\frac{7\pi}{6}$ counterclockwise and measure backward 2 units.

$\left(\text{Note: } \frac{7\pi}{6} = -\frac{5\pi}{6} + 2\pi\right)$

So, all together, polar coord. pairs of P are $\left(2, \frac{\pi}{6} + 2\pi n\right)$, $n = 0, \pm 1, \pm 2, \ldots$ and

$$\left(-2, -\frac{5\pi}{6} + 2\pi n\right),\ n = 0, \pm 1, \pm 2, \ldots$$

$n = 0$: $\quad P\left(2, \frac{\pi}{6}\right)$ & $P\left(-2, -\frac{5\pi}{6}\right)$

$n = 1$: $\quad P\left(2, \frac{13\pi}{6}\right)$ & $P\left(-2, \frac{7\pi}{6}\right)$

$n = -1$: $\quad P\left(2, -\frac{11\pi}{6}\right)$ & $P\left(-2, -\frac{17\pi}{6}\right)$

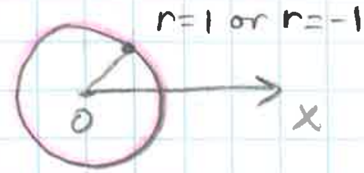
and so on.

<u>Note:</u> $\quad P(r, \theta)$

1) If r is fixed, say, $r = a \neq 0$, θ varies

$\Rightarrow$ P traces a circle of radius $|a|$ centered at O.

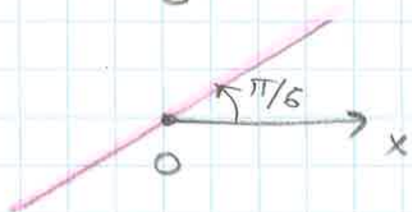
2) If θ is fixed, say, $\theta = \theta_0$, and r varies $(-\infty < r < \infty)$ $\Rightarrow$ points $P(r, \theta_0)$ lie on the line going through the origin.



Example 2

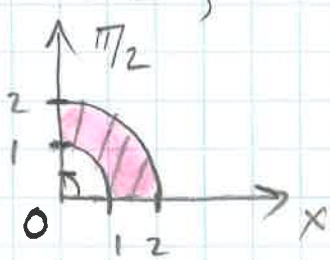
a) $r=1$ & $r=-1$ are equations for the circle of radius 1 centered at O.

b) $\theta = \frac{\pi}{6}$ is an equation for the line

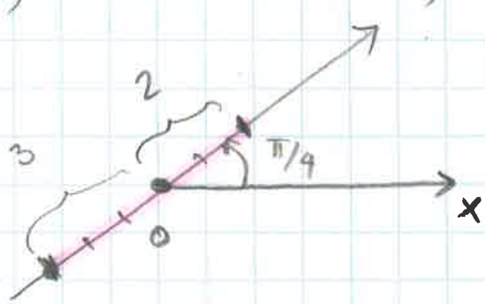


c) Graph the sets of points:

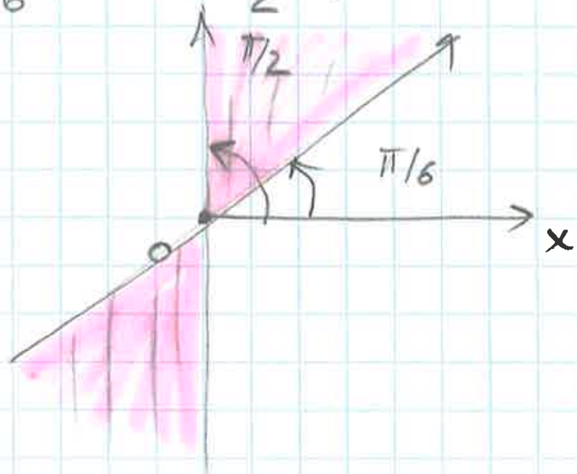
(i) $1 \le r \le 2$, $0 \le \theta \le \frac{\pi}{2}$



(ii) $-3 \le r \le 2$, $\theta = \frac{\pi}{4}$



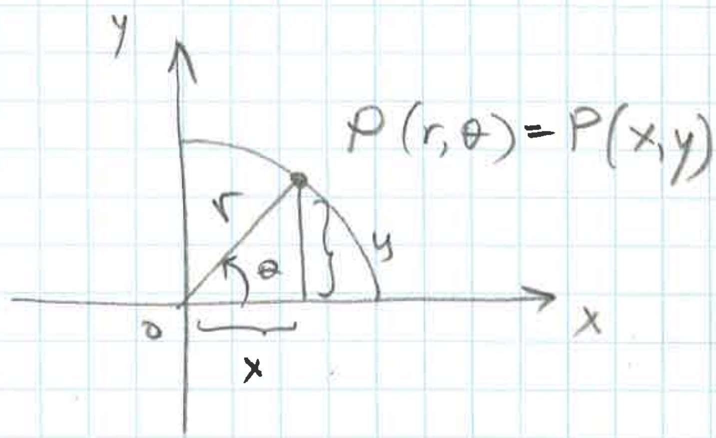
(iii) $\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$, $-\infty < r < \infty$



Relating Polar & Cartesian Coordinates.

initial ray → positive x-axis
ray $\theta = \frac{\pi}{2}$, $r > 0$ → positive y-axis

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ x^2+y^2 &= r^2 \\ \tan\theta &= \frac{y}{x} \end{aligned}}$$



Example 3.

Polar	↔	Cartesian
$r\cos\theta = 2$		$x = 2$
$r^2 \sin\theta\cos\theta = 4$		$xy = 4$
$r^2(\cos^2\theta - \sin^2\theta) = 1$		$x^2 - y^2 = 1$
$r = -8\cos\theta$		$x^2 + y^2 = -8x$

The last one:

$$r = -8\cos\theta \Rightarrow \underbrace{r^2}_{} = -8\underbrace{r\cos\theta}_{} \Rightarrow$$

$$x^2+y^2 = -8x$$
$$\left(\Rightarrow (x+4)^2+y^2=16 \right)$$

Example 4.

Convert $2x-5x^3=1+xy$ into polar coordinates.

$$2(r\cos\theta)-5(r\cos\theta)^3 = 1+(r\cos\theta)(r\sin\theta)$$

$$2r\cos\theta - 5r^3\cos^3\theta = 1+r^2\cos\theta\sin\theta$$

(Read Examples 5, 6 on pp. 576-577)