§ 10.4 Graphing Polar Coordinate Equations.

- Plot in the Cartesian plane
- Use symmetry (see table on p. 578)

about x-axis

(r, θ) & $(r, -\theta)$ or $(-r, \pi-\theta)$

about y-axis

(r, θ) & $(r, \pi-\theta)$ or $(-r, -\theta)$

about the origin

(r, θ) & $(-r, \theta)$ or $(r, \theta+\pi)$

- Slopes can be useful too:

Assume a polar curve is given in the form $r = f(\theta)$. Then

$$\left.\begin{aligned} x &= r\cos\theta = f(\theta)\cos\theta \\ y &= r\sin\theta = f(\theta)\sin\theta \end{aligned}\right\}$$

param. equations w/ parameter θ

Recall: $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} \Rightarrow$ slope

$$\boxed{\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}}$$

provided $\dfrac{dx}{d\theta} \neq 0$

Note: if $r = f(\theta)$ passes through O at $\theta = \theta_0$ then $f(\theta_0) = 0 \Rightarrow$

$$\left.\frac{dy}{dx}\right|_{(0,\theta_0)} = \frac{f'(\theta_0)\sin\theta_0}{f'(\theta_0)\cos\theta_0} = \tan\theta_0$$

Example 1 Cardioid ("heart shape")

$r = 1 - \cos\theta$

Plot several points (r, θ) & connect them as θ increases. Use symmetry.

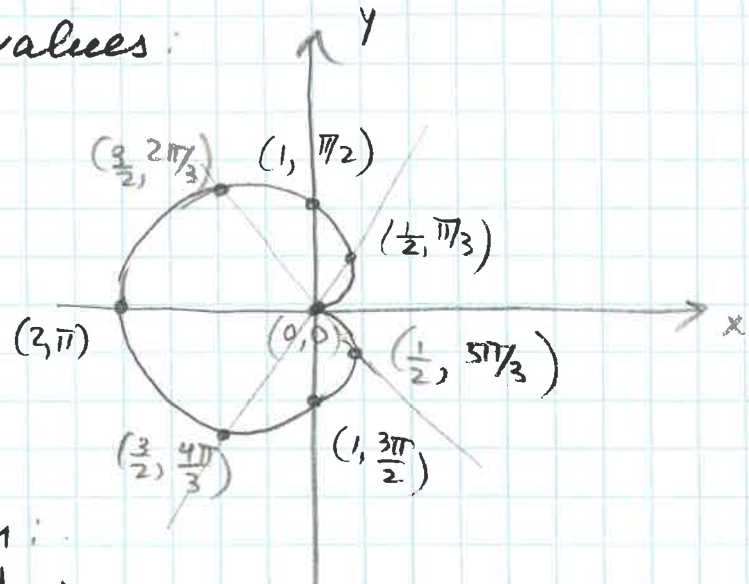
Note: if (r, θ) is on the graph $\Rightarrow$

$r = 1 - \cos\theta = 1 - \cos(-\theta) \Rightarrow (r, -\theta)$ is on the graph.

So, we have symmetry about the x-axis, therefore, we can focus on plotting the upper half and then reflect it across the x-axis; $0 \le \theta \le \pi$.

Table of (r, θ) values:

θ	$r = 1 - \cos\theta$
0	0
$\pi/3$	$1/2$
$\pi/2$	1
$2\pi/3$	$3/2$
π	2



Slopes at the origin:

$$\left.\frac{dy}{dx}\right|_{(0,0)} = \tan 0 = \left.\frac{dy}{dx}\right|_{(0,2\pi)} = \tan 2\pi = 0$$

(Read Examples 2, 3 on pp. 578–580)

Example 2. Determine the equation of the tangent line to $r = 3 + 8\sin\theta$ at $\theta = \frac{\pi}{2}$

$r = 3 + 8\sin\theta \Rightarrow \frac{dr}{d\theta} = f'(\theta) = 8\cos\theta.$

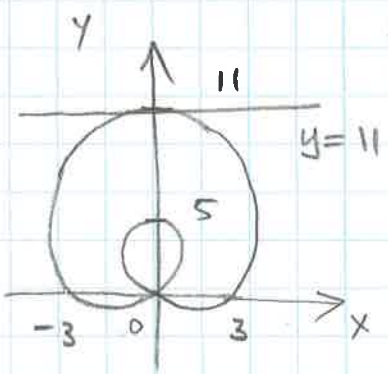
So, $\frac{dy}{dx} = \frac{8\cos\theta\sin\theta + (3+8\sin\theta)\cos\theta}{8\cos^2\theta - (3+8\sin\theta)\sin\theta}$

$= \frac{16\cos\theta\sin\theta + 3\cos\theta}{8\cos^2\theta - 3\sin\theta - 8\sin^2\theta} \Rightarrow$

Slope $m = \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{16\cdot\cos\frac{\pi}{2}\sin\frac{\pi}{2} + 3\cos\frac{\pi}{2}}{8\cos^2\frac{\pi}{2} - 3\sin\frac{\pi}{2} - 8\sin^2\frac{\pi}{2}}$

$= 0 \Rightarrow$ horizontal tangent at $\theta = \frac{\pi}{2}$

$r\left(\frac{\pi}{2}\right) = 3 + 8\sin\frac{\pi}{2} = 11 \Rightarrow$ tangent equation is $\boxed{y = 11}$



(Use the table approach to plot $r = 3 + 8\sin\theta$)