§14.3 Area by Double Integration.

If $f(x,y)=1$ in definition of double integral:

$$S_n = \sum_{k=1}^{n} \Delta A_k \sim \text{sum of areas of the small rectangles covering region } R$$

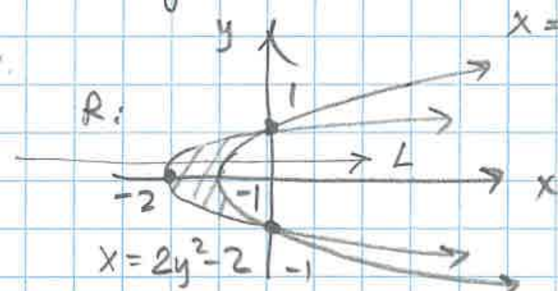
$$\Rightarrow \sum_{k=1}^{n} \Delta A_k \sim \text{area of } R \; !$$

- The area of a closed, bounded plane region R is

$$\boxed{A = \iint_R dA}$$

Example: Find the area between the parabolas $x = y^2 - 1$ & $x = 2y^2 - 2$ by double integration.

Solution:



$2y^2 - 2 = y^2 - 1$

$y^2 = 1$

$y = \pm 1 \Rightarrow x = 0$

pts of intersection

$$A = \iint_R dA = \int_{-1}^{1}\int_{2y^2-2}^{y^2-1} dx\,dy = \int_{-1}^{1} [y^2 - 1 - 2y^2 + 2]\,dy$$

$$= \int_{-1}^{1} (-y^2 + 1)\,dy = \left[-\frac{y^3}{3} + y\right]_{-1}^{1} = \boxed{4/3}$$

Read Examples 1-3 pp. 770-771.

- Average Value of $f(x,y)$ over R

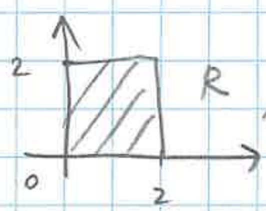
is $\boxed{\dfrac{1}{\text{area of } R}\iint\limits_R f(x,y)\,dA}$

(Recall from Calc I: Aver. value $= \dfrac{1}{b-a}\int_a^b f(x)\,dx$)

Example: Find the average value of

$f(x,y) = x^2+y^2$ over R: $0 \le x \le 2$, $0 \le y \le 2$.

average value $= \dfrac{1}{4}\iint\limits_R (x^2+y^2)\,dA$



$$= \frac{1}{4}\int_0^2\int_0^2 (x^2+y^2)\,dx\,dy = \boxed{8/3}$$