

Mathematica Project 2: due February, 13th (IN CLASS)

Multivariate Calculus, MTH 212, Spring 2019

Note: late projects will not be accepted.

Use the text input mode to start your Mathematica notebook with your name and project number. Your project should be well-organized and clear to read; make sure all the exercises are clearly labeled and all questions answered. (Failure to follow these directions will result in lost points.) Make sure that you get all your Mathematica input (functions, formulas, commands you use to answer questions) and the required output (evaluate all the necessary cells to produce/display your results, plots, etc.). Bring your project printout to class on 2/13/19.

1. Use Mathematica to graph and identify sets of points defined by the following equations (use comment or text input mode to describe your results):

(a) $r = 3 \sec(\theta - \pi/3)$

(b) $r = -2 \cos \theta$

(c) $r = \frac{8}{4 + \sin \theta}$

2. *Hyperbolic Spiral.* This curve, also called *Reciprocal Spiral*, was studied by P. Nicolas in 1696, Varignon in 1704, Bernoulli in 1710, and Cotes in 1722. The general equation is $r = a/\theta$, with a constant. (Note that the hyperbolic spiral is the opposite of an Archimedean spiral $r = a + b\theta$.) It begins at an infinite distance from the pole in the center (for θ starting from zero, $r = a/\theta$ starts from infinity), and it winds faster and faster around as it approaches the pole.

Use Mathematica to plot a spiral for $a = 1$, for θ from some very small positive value (say, 10^{-16}) to 4π (or bigger). (You can try to start from $\theta = 0$, see what happens.) Doesn't the reciprocal spiral remind of the tail of a chameleon?

Next, find the length of the spiral for $1 \leq \theta \leq 4\pi$ setting up the corresponding integral and using Mathematica to compute it.

3. The area of the region that lies inside the cardioid curve $r = \cos \theta + 1$ and outside the circle $r = \cos \theta$ IS NOT

$$\frac{1}{2} \int_0^{2\pi} [(\cos \theta + 1)^2 - \cos^2 \theta] d\theta.$$

Why NOT? (Give an explanation.)

What IS the area? Use Mathematica to plot the curves, set up the correct formula and calculate the area.

4. Use *ContourPlot3D* to help you to plot and give a geometric description of the sets of points in space whose coordinates satisfy the given pairs of equations. For each problem, graph the equations in 3D space, picking a range that gives a good view of all the main features of the graphs. Label the axes using *AxesLabel* so that I won't lose track of the axes as I rotate your plots. (For each plot, use comment or text input mode to give a good geometric description of the sets of points defined by the intersection of the pair of equations.)

(a) $x^2 + y^2 + z^2 = 25$ and $y = -4$

(b) $x^2 + y^2 = 4$ and $z = y$

(c) $z = y^2$ and $x = 1$