Mathematica Project 3: due March, 24th
Multivariate Calculus, MTH 212, Srping 2021

Note: Late projects will not be graded.

Failure to follow directions below may result in lost points.

Use the text input mode to start your Mathematica notebook with your name and project number. Your project should be well-organized and clear to read; make sure all the exercises are clearly labeled and all questions answered (use comments or text mode). Make sure that you get all your Mathematica input (functions, formulas, commands you use to answer questions) and the required output (evaluate all the necessary cells to see your results, plots, etc.). While working on the project, you may want to use the help file posted on our web page. Once the project is completed, review it and submit it to the appropriate folder at `http://LIVE.wilkes.edu` anytime before 11:59 pm on 3/24/21. *The name of your .nb file should identify you clearly.* (A good example of a name could be John_Smith_Project3.nb.) For an easier upload of your file, you may **delete all output from your notebook** - you can find the corresponding command under "Cell" in your notebook top panel.

1. *Hyperbolic Spiral.* This curve, also called *Reciprocal Spiral*, was studied by P. Nicolas in 1696, Varignon in 1704, Bernoulli in 1710, and Cotes in 1722. The general polar equation is $r = a/\theta$, with a constant. It begins at an infinite distance from the pole in the center (for θ starting from zero, $r = a/\theta$ starts from infinity), and it winds faster and faster around as it approaches the pole.
   Use Mathematica to plot a spiral for $a = 1$, for θ from some very small positive value (say, 10^{-16}) to 4π (or bigger). (You can try to start from $\theta = 0$, see what happens.) Doesn't the reciprocal spiral remind of the tail of a chameleon?
   Next, find the length of the spiral for $1 \leq \theta \leq 4\pi$ setting up the corresponding integral and using Mathematica to compute it.

2. The area of the region that lies inside the cardioid curve $r = \cos\theta + 1$ and outside the circle $r = \cos\theta$ IS NOT

$$\frac{1}{2}\int_0^{2\pi} \left[(\cos\theta + 1)^2 - \cos^2\theta\right] d\theta.$$

Why NOT? (Give an explanation.)

What IS the area? Use Mathematica to plot the curves in one figure, set up the correct formula and calculate the area.

3. For each function given below in (i)-(iii), perform the following steps in Mathematica (do not forget to comment/text your steps):

   (a) Plot the surface $z = f(x, y)$ (you can use either *ContourPlot3D* or *Plot3D*, make sure to use the proper parameters in either case, use "Help"). Choose appropriate range for the variables so that the graph features are clearly seen in the figure.

   (b) Plot several level curves $f(x, y) = c$ for $c = \{-2, -1, 0, 1, 2\}$, using *ContourPlot*: explore the options of *ContourPlot* to *have all the level curves in one figure.*

   (i) $f(x, y) = \sin^2 x - 4\cos y$

   (ii) $f(x, y) = e^{-y}\cos x$

   (iii) $f(x, y) = 2y^2 + 2y^3 - 2x^3$

4. We cannot plot the 4D graph (x, y, z, w) of a function of three variables $w = f(x, y, z)$, but we can look at its 3D level surfaces $f(x, y, z) = c$. Given the function of three variables $w = z - x^2 - y^2$, choose five distinct constants c_1, c_2, c_3, c_4, and c_5 to plot five level surfaces $z - x^2 - y^2 = c_i$ $(i = 1, 2, 3, 4, 5)$, *in one figure*, using *ContourPlot3D*. To create a nice picture, play with the option *ContourStyle* $\rightarrow$ *Opacity* to make all the surfaces clearly seen in the combined plot.