Mathematica Project 4: due November 4th
Multivariate Calculus, MTH 212, Fall 2021

Note: Late projects will not be accepted.

Failure to follow directions below may result in lost points.

Use the text input mode to start your Mathematica notebook with your name and project number. Your project should be well-organized and clear to read; make sure all the exercises are clearly labeled and all questions answered (use comments or text mode). Make sure that you get all your Mathematica input (functions, formulas, commands you use to answer questions) and the required output (evaluate all the necessary cells to see your results, plots, etc.). Once the project is completed, review it and submit it to the appropriate folder at `http://LIVE.wilkes.edu` **anytime before 11:59 pm on 11/4**. *The name of your .nb file should identify you clearly.* (A good example of a name could be John_Smith_Project4.nb.) For an easier upload of your file, you may **delete all output from your notebook** - you can find the corresponding command under "Cell" in your notebook top panel.

1. From Section 13.5 we know that "At every point (x_0, y_0) in the domain of a differentiable function $f(x, y)$, the gradient vector of f is normal to the level curve through the point (x_0, y_0)." Let us illustrate this using the function $f(x, y) = x^2 - xy + y^2$. In Mathematica, plot the level curve $x^2 - xy + y^2 = 7$ in the xy-plane together with the gradient vector ∇f and the tangent line at the point $(-1, 2)$ (using *Show*). Do necessary calculations (for example, the gradient vector). You can use *Grad* to compute ∇f, but then do not forget to evaluate it at the point $(-1, 2)$. To find the tangent line to the level curve, use Equation (6) on page 741 of the textbook.

2. Consider the level surface $x^2 + y^2 + z = 4$. Use Mathematica to create a 3D plot showing the level surface, the normal line, and the tangent plane to the surface at the point $(1, 1, 2)$ (all in one fugure, use *Show*). Do necessary calculations, such as gradient vector, to obtain equations of the line and plane given by Formulas (1) and (2) on page 745 of the textbook.)

   For help with parts 1 and 2, use the demo notebook on gradients and tangent planes.

3. Find all local extrema and saddle points of the function $f(x, y) = 4xy - x^4 - y^4$. Do all the necessary calculations in Mathematica: finding partial derivatives and Hessian, solving equations for critical points, evaluating the Hessian and f_{xx} at the critical points. Your results should contain a list of the max/min/saddle values of f and the corresponding list of points at which these values are attained. Show all your work. Plot these points on the surface $z = f(x, y)$. *For bonus points, plot some level curves of f in the xy-plane together with the critical points.*

   For help, use the notebook on min/max/saddle points.