Mathematica Project 4: due April, 3rd
Multivariate Calculus, MTH 212, Spring 2020

Note: Late projects will not be accepted.

Failure to follow directions below may result in lost points.

Use the text input mode to start your Mathematica notebook with your name and project number. Your project should be well-organized and clear to read; make sure all the exercises are clearly labeled (use comment or text mode) and all questions answered. Make sure that you get all your Mathematica input (functions, formulas, commands you use to answer questions) and the required output (evaluate all the necessary cells to produce/display your results, plots, etc.). While working on the project, you may want to use the help file posted on our web page.

Once the project is completed, review it and submit it to the appropriate folder at `http://LIVE.wilkes.edu` at anytime before 11:59 pm on 4/3/20.
The name of your .nb file should identify you clearly. (A good example of a name could be John_Smith_Project4.nb.) Before submitting, **delete all output from your notebook** - you can find the corresponding command under "Cell" in your notebook top panel.

1. From Section 13.5 we know that "At every point (x_0, y_0) in the domain of a differentiable function $f(x, y)$, the gradient vector of f is normal to the level curve through the point (x_0, y_0)." Let us illustrate this using the function $f(x, y) = x^2 - xy + y^2$. In Mathematica, plot the surface $z = f(x, y)$. Then plot the level curve $x^2 - xy + y^2 = 7$ in the xy-plane together with the gradient vector ∇f and the tangent line at the point $(-1, 2)$ (using *Show*). Do necessary calculations. You can use *Grad* to compute ∇f, but then do not forget to evaluate it at the point $(-1, 2)$. To find the tangent line to the level curve, use Equation (6) on page 717 of the textbook.

2. Consider the surface $x^2 + y^2 + z = 4$. Use Mathematica to create a 3D plot showing the graph of the surface, the normal line, and the tangent plane to the surface at the point $(1, 1, 2)$ (all in one fugure, use *Show*). Do necessary calculations, such as gradient

vector, to obtain equations of the line and plane given by Formulas (1) and (2) on page 721 of the textbook.)

For help with parts 1 and 2, use the demo notebook on gradients and tangent planes.

3. *Lagrange multipliers.* Find the maximum and mimimum values of $f(x, y) = x^2 + y^2$, subject to the constraint $g(x, y) = x^2+xy+y^2-1 = 0$. Do all the necessary calculations in Mathematica: finding partial derivatives, solving equations, evaluating the objective function at the critical points. Your results should contain the maximum and minimum values of f and the points at which these values are attained. Show all your work. Choose one point of extremum and plot (in the xy-plane) a level curve of f going through this point together with the constraint curve $g(x, y) = 0$ and the gradient vectors of the functions f and g there.

   For help, use the Project 4 Help file posted on our web page.