Mathematica Project 4: due April, 3rd (IN CLASS)
Multivariate Calculus, MTH 212, Spring 2019

Note: late projects will not be accepted.

Use the text input mode to start your Mathematica notebook with your name and project number. Your project should be well-organized and clear to read; make sure all the exercises are clearly labeled and all questions answered. (Failure to follow these directions will result in lost points.) Make sure that you get all your Mathematica input (functions, formulas, commands you use to answer questions) and the required output (evaluate all the necessary cells to produce/display your results, plots, etc.). Bring your project printout to class on 4/3/19. (Please staple it!)

1. For each function given below in (i)-(iii), perform the following steps in Mathematica (do not forget to comment/text your steps in the notebook):

   (a) Plot the surface $z = f(x, y)$.

   (b) Plot several level curves $f(x, y) = c$ for $c = -2, -1, 0, 1, 2$, using *ContourPlot*: explore the options of *ContourPlot* to have all the level curves in one figure.

   (i) $f(x, y) = \sin x - \cos y$

   (ii) $f(x, y) = e^{-y} \cos x$

   (iii) $f(x, y) = 2y^2 - 2y^4 + 2x^2$

2. We cannot plot the 4D graph of a function of three variables $w = f(x, y, z)$, but we can look at its 3D level surfaces $f(x, y, z) = c$. Given the function of three variables $w = z - x^2 - y^2$, choose three constants c_1, c_2, and c_3 to plot three level surfaces $z - x^2 - y^2 = c_i$ $(i = 1, 2, 3)$, in one figure, using *ContourPlot3D.* To create a nice picture, play with the option *ContourStyle* $\to$ *Opacity* to make all three surfaces clearly seen in the combined plot.

3. From Section 13.5 we learn that "At every point (x_0, y_0) in the domain of a differentiable function $f(x, y)$, the gradient vector of f is normal to the level curve through the point

(x_0, y_0)."

Let us illustrate this using the function $f(x, y) = x^2 - xy + y^2$.

In Mathematica, plot the surface $z = f(x, y)$. Then plot the level curve $x^2 - xy + y^2 = 7$ in the xy-plane together with the gradient vector ∇f and the tangent line at the point $(-1, 2)$ (plot them together using *Show*). Show necessary calculations. You can use *Grad* to compute ∇f, but then do not forget to evaluate it at the point $(-1, 2)$. To find the tangent line to the level curve, use Equation (6) on page 717 of the textbook.

4. Consider the surface $x^2 + y^2 + z = 4$. Use Mathematica to create a 3D plot showing the graph of the surface, the normal line, and the tangent plane to the surface at the point $(1, 1, 2)$. (Do the necessary calculations, such as gradient vector – either by hand or using Mathematica – to obtain the equations of the line and plane given by Formulas (1) and (2) on page 721 of the textbook.)