Mandelbrot Set and Julia Sets

CS/MTH 364/464

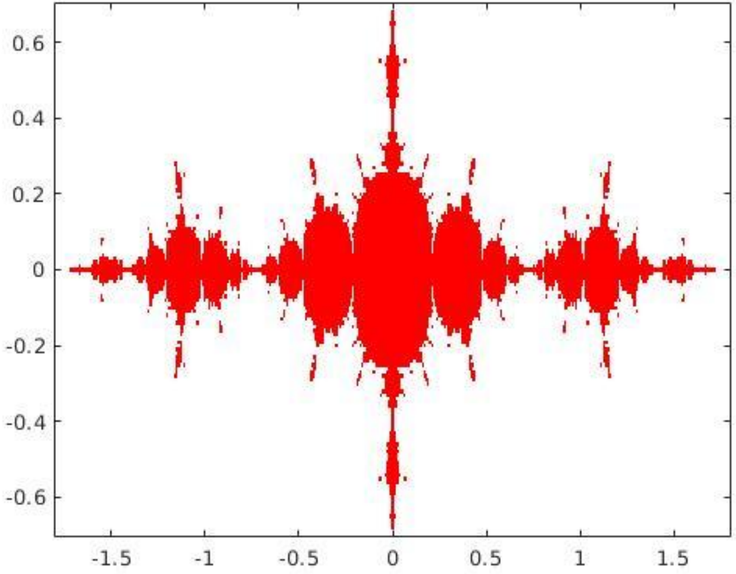
Spring 2023

(Filled) Julia Set

Set of all initial guesses z_0 for which the orbit z_0 under the map $z_n{}^2 + c$ remains bounded

$$z_{n+1} = z_n^2 + c$$

Let c = -1.25

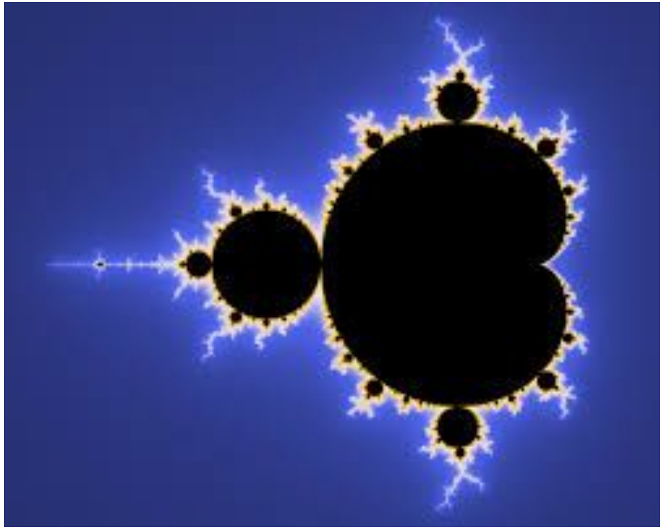


Gaston Julia, 1893-1978

Mandelbrot set

$$z_{n+1} = z_n^2 + c$$

Same equation, but now we study c's



z_n and **c** are complex numbers

Benoit Mandelbrot created the fractal set that consists of all numbers c in the complex plane for which the equation above generates a ***bounded orbit*** $z_0, z_1, z_2, z_3, z_4, \ldots,$ **starting from z_0 = 0**
(the corresponding Julia sets are connected)

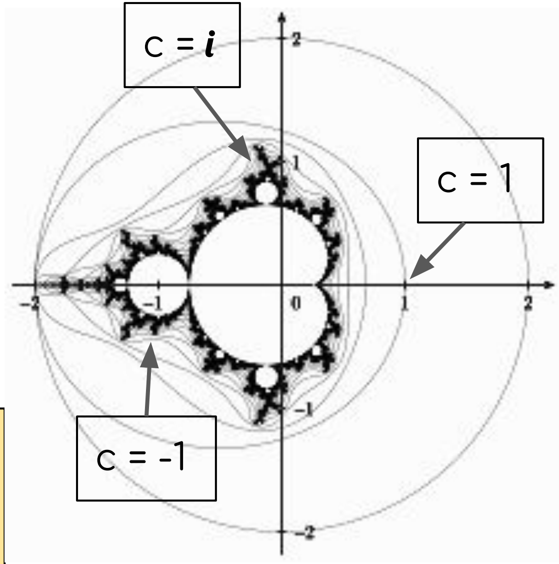
Mandelbrot set

$$z_{n+1} = z_n^2 + c$$

Choose $c = 1$. Then:
$z_0 = 0$, $z_1 = 1$, $z_2 = 2$, $z_3 = 5$, $z_4 = 26$, ... ⇒ z_n grows without bound!
So, $c = 1$ is NOT in the Mandelbrot set! ($c = 1$ is escaping the set.)

Choose $c = -1$. Then:
$z_0 = 0$, $z_1 = -1$, $z_2 = 0$, $z_3 = -1$, $z_4 = 0$, ... ⇒ z_n is either -1 or 0 ⇒ bounded.
So, $c = -1$ is in the Mandelbrot set.

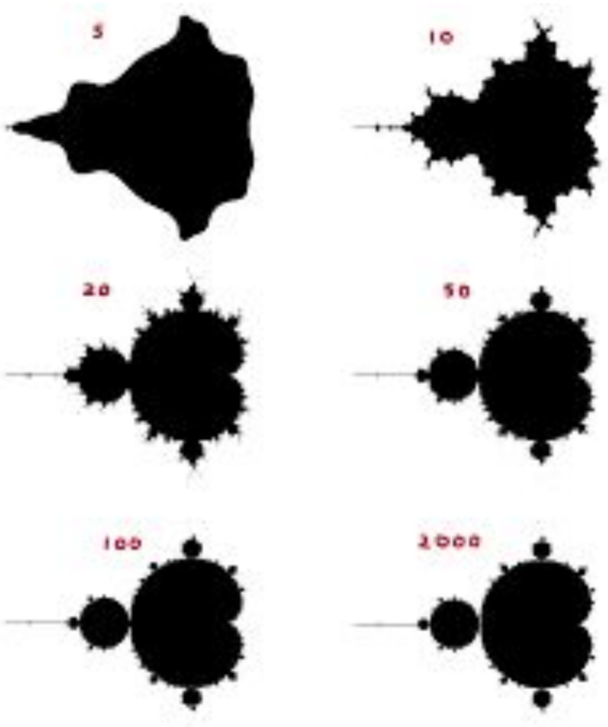
Choose $c = i$. Then:
$z_0 = 0$, $z_1 = i$, $z_2 = -1 + i$, $z_3 = -i$, $z_4 = -1 + i$, $z_5 = -i$, ... ⇒ z_n is either $-i$ or $-1 + i$ ⇒ bounded. So, $c = i$ is in the Mandelbrot set.

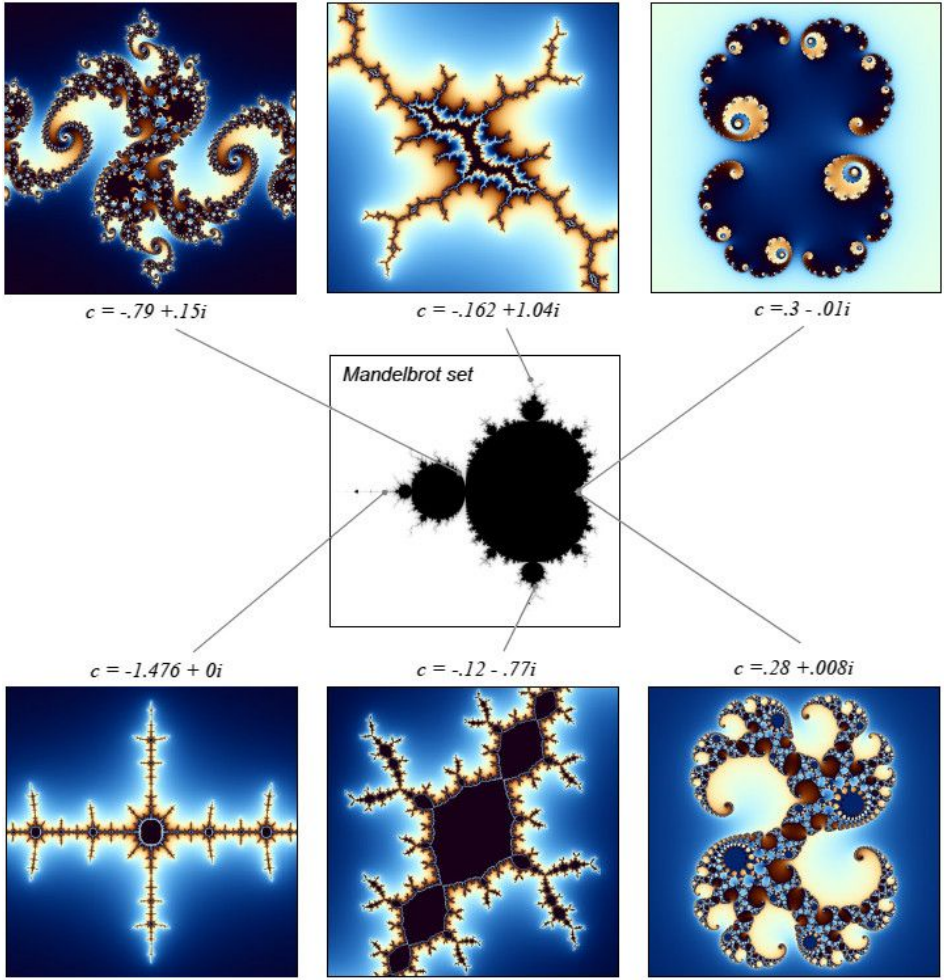


Mandelbrot set

$$z_{n+1} = z_n^2 + c$$

Mandelbrot sets created for different number of iterations: n = 5, 10, 20, 50, 100, 2000.





Julia Sets vs Mandelbrot Set

A specific Julia set can be defined by a point in the Mandelbrot set matching its constant c value, and the look of an entire Julia set is usually similar in style to the Mandelbrot set at that corresponding location.

Points near the edges of the Mandelbrot set typically give the most interesting Julia sets.

Julia Sets and Mandelbrot Set are fractals!

The term "**fractal**" (1975) was coined by the mathematician Benoit Mandelbrot, based on the Latin *fractus*, meaning *"broken"* or *"fractured"*.

"A fractal is a way of seeing infinity".

Benoit Mandelbrot
1924 – 2010

Fractal mathematics or **fractal geometry** is one of the most important discoveries of the 20th century.
It has tools that allow us to describe *clouds, trees, coastlines, romanesco broccoli, and other objects that cannot be described using traditional geometry.*

Fractals have 2 important features: self-similarity and fractal (non-integer) dimension.