§10.4 Clenshaw – Curtis Quadrature.

- Newton-Cotes: - equally-spaced $n+1$ nodes $x_0, \ldots, x_n$
  - integrate $p(x)$ that interpolate the function $f(x)$ at these nodes.

- Gauss Quadrature: - $n+1$ nodes are chosen to be zeroes of an orthogonal polynomial $q(x)$ of deg. $n+1$ s.t. the approx. formula is exact for $p(x)$ of deg. $2n+1$.

- Clenshaw – Curtis Quadrature: – uses Chebyshev points to interpolate $p(x)$ and then integrate

Recall: $x_j = \cos\left(\frac{\pi j}{n}\right)$, $j = 0, 1, \ldots, n$, on $[-1, 1]$.

(Note: if $[a,b] \neq [-1,1]$ then use transformation $\ell : [-1,1] \to [a,b]$ w/ $\ell(x) = \frac{a+b}{2} + \left(\frac{b-a}{2}\right)x$.)

The method requires less work, but can provide even better approximation then Gauss quadrature. Also, the approximation formula is exact for polynomials of degree n or less.

Note: Gauss quadrature has $O(n^2)$ FLOPs, while Clenshaw-Curtis formula can be implemented w/ Fast Fourier Transform (FFT) §14.5.1 w/ $O(n \ln n)$ FLOPs.

(use "chebfun" package)