Mathematica Project 4: due April, 5th, at 11:59pm
Multivariate Calculus, MTH 212, Spring 2018

Turn in the project by uploading your Mathematica notebook to the appropriate folder at `http://LIVE.wilkes.edu` at anytime before 11:59 pm on Wednesday, 4/5/18.
The name of your .nb file should identify you clearly. (A good example of a name could be John_Smith_Project2.nb.)

Inside your notebook:

Use the text input mode to start your notebook with your name and project number.

Make sure all the problems are clearly labeled and all questions answered. (Failure to follow these directions will result in lost points.) Make sure that you get the required output by evaluating the notebook, but before submitting, **delete all output from your notebook** - you can find the corresponding command under "Cell" in your notebook top panel.

Note: sorry, but late projects will not be accepted.

1. (30pts) For each function given below, perform the following steps in Mathematica (do not forget to annotate your graphs and comment/text your steps in the notebook):

   (a) Plot the surface $z = f(x, y)$.

   (b) Plot several level curves $f(x, y) = c$ for $c = -5, -1, 0, 1, 5$, using *ContourPlot*. Explore the options of *ContourPlot* to have all the level curves in one plot and annotate them with legends of contour regions.

   i. $f(x, y) = \sin x - \cos y$

   ii. $f(x, y) = e^{-y} \cos x$

   iii. $f(x, y) = 2y^2 - 2y^4 + 2x^2$

2. (10pts) We cannot plot the 4D graph of a function of three variables, but we can look at its 3D level surface. Given the function of three variables $w = z - x^2 - y^2$, choose three constants c_1, c_2, and c_3 to plot three level surfaces $z - x^2 - y^2 = c_i$, $i = 1, 2, 3$,

in one figure, using *ContourPlot3D*. To create a nice picture, play with the option *ContourStyle* $\rightarrow$ *Opacity* to make all three surfaces clearly seen in the combined plot.

3. (25pts) From Section 13.5 we know that "At every point (x_0, y_0) in the domain of a differentiable function $f(x, y)$, the gradient vector of f is normal to the level curve through the point (x_0, y_0)."
   Let us illustrate this using the function $f(x, y) = x^2 - xy + y^2$.
   In Mathematica, plot the surface $z = f(x, y)$. Then plot the level curve $x^2 - xy + y^2 = 7$ in the xy-plane together with the gradient vector ∇f and the tangent line at the point $(-1, 2)$ (plot them together using *Show*). Use text cell or comment mode to explain your steps and necessary calculations. You can use *Grad* to compute ∇f, but then do not forget to evaluate it at the point. To find the tangent line to a level curve, use Equation (6) on page 717.

4. (25pts) Consider the surface $x^2 + y^2 + z = 4$. Use Mathematica to create a 3D plot showing the graph of the surface, the normal line, and the tangent plane to the surface at the point $(1, 1, 2)$. (Do the necessary calculations, such as gradient vector – either by hand or using Mathematica – to obtain the equations of the line and plane given by Formulas (1) and (2) on page 721, Section 13.6.)