MTH 331/331&H/431 STUDY GUIDE FOR EXAM I – FALL 2021

Exam I will be held on Monday, September 27th (usual time and room). You can use a one-sided letter size cheat-sheet.

Important things to understan/know/apply:

- Set theory (section 1.2):
  - Sets: definitions and relations (*e.g.*, Proposition 1.2, De Morgan's laws).
  - Cartesian products and mappings; definitions and examples of injective, surjective, and bijective maps, composition and inverse maps.
  - Equivalence relations and partitions: definitions and examples.
- Integers (sections 2.1 and 2.2):
  - Use mathematical induction to prove statements.
  - Well-ordered sets. The principle of well-ordering.
  - The division algorithm.
  - Use the Euclidean algorithm to find the greatest common divisor of two integers a and b, $d = gcd(a, b)$ and find integers r and s such that $ar + bs = d$. Relatively prime integers.
  - Prime numbers: definition, Euclid's lemma and theorem, Fundamental Theorem of Arithmetic.
- Groups (sections 3.1 and 3.2):
  - Integers modulo n; addition and multiplication modulo n. Symmetries.
  - Definition of a group. Abelian (or commutative) groups. Basic properties of groups. Examples of groups.
  - Given a set with a binary operation, show whether it is a group or not, using the definition (closure + three axioms!). Cayley tables.
  - Uniqueness of the identity and inverse elements. Shoes-Socks property: $(ab)^{-1} = b^{-1}a^{-1}$ and the property $(a^{-1})^{-1} = a$. Right and left cancellation laws. Laws of exponents for multiplicative and additive groups.