MTH 212 - MULTIVARIATE CALCULUS - STUDY GUIDE FOR EXAM II

No books, notes, calculators, or cell phones are permitted during the test!

Use notes, text, homework, and suggested exercises to prepare for the test.

- Chapter 11: "Vectors and the Geometry of Space".
  - Section 11.2: *Vectors.*
    - All the basics: definition of a vector, component form of a vector, magnitude (length) of a vector, addition (difference) and multiplication by a scalar, properties of vector operations, unit vectors, standard unit vectors, midpoint of a line segment.
  - Section 11.3: *The Dot Product.*
    - The dot product formulas: $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 = |\mathbf{u}||\mathbf{v}| \cos\theta$.
      (Also: review the properties of the dot product!)
    - Angle between two vectors: $\theta = \cos^{-1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$.
    - Vector projection: $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}\right)\left(\frac{\mathbf{v}}{|\mathbf{v}|}\right) = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$.
      (The scalar component of $\mathbf{u}$ in the direction of $\mathbf{v}$ is $\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}$.)
    - Work done by a constant force $\mathbf{F}$ in the direction of displacement $\mathbf{D}$ is $W = \mathbf{F} \cdot \mathbf{D}$.
  - Section 11.4: *The Cross Product.*
    - The cross product formulas: $\mathbf{u} \times \mathbf{v} = (|\mathbf{u}||\mathbf{v}| \sin\theta)\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} =$
      $(u_2v_3 - v_2u_3)\mathbf{i} - (u_1v_3 - v_1u_3)\mathbf{j} + (u_1v_2 - v_1u_2)\mathbf{k}$.
      (Also: review the properties of the cross product!)
    - Areas of a triangle ($(1/2)|\mathbf{u} \times \mathbf{v}|$) and parallelogram ($|\mathbf{u} \times \mathbf{v}|$) determined by vectors $\mathbf{u}$ and $\mathbf{v}$.
    - Volume of a parallelepiped determined by vectors vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ via triple scalar (or box) product: $= |(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|$, where $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_2 \end{vmatrix}$.
  - Section 11.5: *Lines and Planes in Space.*
    - Vector and parametric equations for a line in space (need a point on the line, $P_0(x_0, y_0, z_0)$, and a vector $\mathbf{v}$ the line is parallel to). Note: parameterizations are not unique!
    - Distance from point S to a line passing through point P, parallel to $\mathbf{v}$: $d = \frac{|\mathbf{PS}\times\mathbf{v}|}{|\mathbf{v}|}$. (If the line is given to you in a parametric form, you can find point P on the line by plugging in any value of t, e.g., $t = 0$.)
    - For a plane in space: need a point on the plane, $P_0(x_0, y_0, z_0)$, and a vector normal (i.e., orthogonal) to the plane, $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$. The equation is $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$. (One way to get the normal $\mathbf{n}$ is take the cross product of two vectors in the plane (that have the same initial point).)

* Distance from a point to a plane: given point S in space, and a plane with normal $\mathbf{n}$ and point P on the plane, the distance from S to the plane is $d = \frac{|\mathbf{PS}\cdot\mathbf{n}|}{|\mathbf{n}|}$.
* Line of intersection of two planes: given two planes with normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$, respectively, the vector $\mathbf{n}_1 \times \mathbf{n}_2$ points in the direction of the line of intersection of the two planes (assuming they intersect). To get a point on this line, you can solve the system of two plane equations. Note that there will be 3 variables and 2 equations in this linear system, so you should just set one of the variables to a constant, e.g., $x = 0$, and solve for the other two. (Two planes are parallel – don't intersect – if they have parallel normals, i.e., $\mathbf{n}_1 \times \mathbf{n}_2 = \mathbf{0}$.)
* The angle between two planes is the angle between their normal vectors.

- Chapter 12: "Vector Functions and Motion in Space".
  - Sections 12.1 and 12.2: *Position, velocity, and acceleration.*
    * If a vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$ represents the position of a particle moving along a curve, then $\mathbf{v}(t) = \mathbf{r}'(t)$ is the velocity, $|\mathbf{v}(t)|$ is speed, and $\mathbf{a}(t) = \mathbf{v}'(t)$ is the acceleration. If you are given an initial value problem (e.g., $\mathbf{a}(t)$ and initial values of $\mathbf{v}(t_0)$ and $\mathbf{r}(t_0)$ at some point t_0), you integrate to work your way up to $\mathbf{r}(t)$, using the values of $\mathbf{v}(t_0)$ and $\mathbf{r}(t_0)$ to find the constants of integration.
    * Review the differentiation rules for vector functions (p. 647).
    * If $\mathbf{r}$ is a differentiable vector function of constant length ($|\mathbf{r}| = c$), then $\mathbf{r}\cdot\mathbf{r}' = 0$.
  - Section 12.3: *Arc length in Space.*
    * If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ gives a curve in space, the arc length from the point at time $t = a$ to the point at time $t = b$ is

$$\int_a^b |\mathbf{v}(t)|\,dt = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt.$$

      (The arc length parameter $s(t) = \int_a^t |\mathbf{v}(\tau)|\,d\tau$ can be used to reparameterize the curve.)