

MTH 331/431 STUDY GUIDE FOR EXAM III – FALL 2017

Exam III will be held on Friday, November 17th, usual time and room. You can use a one-sided letter size cheat-sheet. We will have a review session during our Thursday class, on November 16th.

Below is a list of important things to understand/know. You will be asked to do small proofs and give definitions.

- Chapter 6: Cosets and Lagrange's Theorem (sections 6.1 - 6.3):
 - What are left and right cosets of a subgroup? Definition and examples.
 - Lemma 6.3: five equivalent statements about cosets.
 - Theorem 6.4: a group G is the disjoint union of the left (right) cosets of a subgroup in G .
 - Theorem 6.8: the number of left cosets of a subgroup is the same as the number of right cosets.
 - Proposition 6.9: number of elements in a subgroup is the same as the number of elements in a coset of this subgroup.
 - Definition of the index of a subgroup H in a group G , $[G : H]$. Lagrange's theorem: $[G : H] = |G|/|H|$. Use the corollaries (e.g., (6.11) the order of an element of a finite group must divide the order of the group; (6.12) if the order of a group is a prime number, then the group is cyclic, and any nontrivial element is a generator).
 - Theorem 6.16 about two cycles of the same length.
 - Fermat's and Euler's theorems (6.18 and 6.19).
- Chapter 9: Isomorphisms and Direct Products (sections 9.1 and 9.2):
 - What is an isomorphism? Definition and examples of isomorphic and non-isomorphic groups.
 - Theorem 9.6 on properties of isomorphisms.
 - Show that any cyclic group of infinite order is isomorphic to \mathbb{Z} , and that any cyclic group of order n is isomorphic to \mathbb{Z}_n . Also, if $|G| = p$, a prime number, then $G \cong \mathbb{Z}_p$.
 - Isomorphism as an equivalence relation: two groups are considered to be the same if they are isomorphic (Theorem 9.10).
 - Cayley's theorem: every group is isomorphic to a group of permutation.
 - What is an external direct product? Definition and examples.
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 - Order of an element in an external direct product (Theorem 9.17).
 - Theorem 9.21: $\mathbb{Z}_{mn} \cong \mathbb{Z}_m \times \mathbb{Z}_n$ iff $\gcd(m, n) = 1$.
 - Theorem 9.27: the internal direct product of subgroups is isomorphic to the external direct product of them.
- Chapter 10: Normal Subgroups and Factor Groups (section 10.1):
 - What is a normal subgroup? Definition and examples. Use Theorem 10.3 to show a subgroup is normal.
 - What is a factor (or quotient) group? Definition and examples. Theorem 10.4.