MTH 212 - MULTIVARIATE CALCULUS - STUDY GUIDE FOR EXAM III

No books, notes, calculators, or cell phones are permitted during the test!

Use notes, text, homework, and suggested exercises to prepare for the test.

Chapter 12: "Vector Functions and Motion in Space".

- Section 12.4 and 12.5: *Curvature, Arc Length Parameter, Unit Tangent and Normal Vectors, Acceleration Decomposed.*
  - $\mathbf{T} = \mathbf{v}/|\mathbf{v}|$ (unit tangent vector)
  - $\mathbf{N} = (d\mathbf{T}/dt)/|d\mathbf{T}/dt|$ (principal unit normal vector)
  - $\mathbf{B} = \mathbf{T} \times \mathbf{N}$ (binormal vector)
  - $\kappa = (1/|\mathbf{v}|)|d\mathbf{T}/dt|$ (curve curvature).
  - Acceleration decomposed: $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$, where $a_T = d/dt(|\mathbf{v}|)$ and $a_N = \sqrt{|\mathbf{a}|^2 - a_T^2}$ are the tangential and normal components of acceleration.

Chapter 13: "Partial Derivatives".

- Section 13.1 and 13.2: *Functions of Several Variables. Limits and Continuity.*
  - Domain and range of functions of two and three variables, $z = f(x, y)$ and $w = f(x, y, z)$. Level curves $f(x, y) = c$ and level surfaces $f(x, y, z) = c$.
  - Basic limit computations. Two-path test for non-existence of a limit.
  - Continuity: $f(x, y)$ is continuous at a point (x_0, y_0) if $\lim_{(x,y)\to(x_0,y_0)} f(x, y) = f(x_0, y_0)$.

- Section 13.3: *Partial Derivatives.*
  - Partial derivatives f_x, f_y (and f_z, if f is a function of x, y, z). If f_x and f_y are continuous at (a, b) then $f(x, y)$ is differentiable, and, therefore, continuous at (a, b).
  - For $f(x, y)$ continuous together with its first and second partial derivatives at a point (a, b), $f_{xy}(a, b) = f_{yx}(a, b)$ (The Mixed Derivative Theorem).
  - Implicit partial differentiation.

- Sections 13.4: *The Chain Rule.*
  - Multivariate chain rule: draw the branch diagram of dependencies and trace from the function at the top to the desired variable at the bottom in each possible way. (Know how to write a general formula for a derivative using the chain rule and use the formula to find the derivative of a prticular function.)
  - Revisited implicit differentiation using the multivariate chain rule: given $F(x, y) = 0$, $dy/dx = -F_x/F_y$; given $F(x, y, z) = 0$, $\partial z/\partial x = -F_x/F_z$ and $\partial z/\partial y = -F_y/F_z$.

- Section 13.5: *Directional Derivatives and Gradient Vectors.*
  - The derivative of the function f in the direction of a unit vector $\mathbf{u}$ at the point P_0 is $D_{\mathbf{u}}f(P_0) = \nabla f(P_0) \cdot \mathbf{u}$. (Make sure $\mathbf{u}$ is a unit vector, if not, divide it by the length.) ∇f is the gradient vector of f (the vector of partial derivatives).
  - The direction of the greatest increase of f is $\mathbf{u} = \nabla f$ and $D_{\mathbf{u}}f(P_0) = |\nabla f(P_0)|$. The direction of the greatest decrease of f is $\mathbf{u} = -\nabla f$ and $D_{\mathbf{u}}f(P_0) = -|\nabla f(P_0)|$. The function f has no change in the direction $\mathbf{u}$ normal to ∇f, i.e. $D_{\mathbf{u}}f(P_0) = 0$.
  - At every point in the domain of f, the gradient ∇f is normal to the level curve going through the point.
- Section 13.6 *Tangent Planes.*
  - The equation of the plane tangent to the level surface $f(x, y, z) = c$ at a point $P_0(x_0, y_0, z_0)$ is $f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0) = 0$.
    Normal line to the surface at the point P_0 has parametric equations $x = x_0 + f_x(P_0)t$, $y = y_0 + f_y(P_0)t$, $z = z_0 + f_z(P_0)t$.
  - The equation of the plane tangent to the surface given by $z = f(x, y)$ at a point $P_0(x_0, y_0, z_0)$ is $f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) - (z - z_0) = 0$ where $z_0 = f(x_0, y_0)$.
  - The linearization of $f(x, y)$ at a point (x_0, y_0) is $L(x, y) = f(P_0) + f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0)$.
- Section 13.7 *Extreme Values and Saddle Points.*
  - Finding the critical points of $f(x, y)$, set all partials to 0, $f_x = f_y = 0$, and find all solutions. (We also have a critical point if f_x, f_y do not exist there.) For each critical point, compute the Hessian $H = f_{xx}f_{yy} - f_{xy}^2$. If $H < 0$ then this is a saddle point. For $H > 0$, if $f_{xx} > 0$ then this is a local minimum and if $f_{xx} < 0$ then this is a local maximum. If $H = 0$, the test is inconclusive.
  - Finding the absolute max and min of f over a bounded region R:
    1) First find all the critical points of f, discarding any points not in R.
    2) Then find all critical points along each portion of the boundary and find local max/min values.
    3) Finally, for all points found in 1) and 2), choose the absolute max and the min values of f.