MTH 212 - MULTIVARIATE CALCULUS - STUDY GUIDE FOR EXAM IV

No books, notes, calculators, or cell phones are permitted during the test!

Use notes, text, homework, and suggested exercises to prepare for the test.

Chapter 14 "Multiple Integrals" and Chapter 15 "Integrals and Vector Fields"

- Section 14.1: *Double and Iterated Integrals over Rectangles.*

  Evaluate a double integral of f over a rectangular area R, $\int\int_R f(x,y)\ dA$, using the iterated (repeated) integral $\int_a^b \int_c^d f(x,y)\ dydx$ or $\int_c^d \int_a^b f(x,y)\ dxdy$ (First Form of Fubini's Theorem). For a positive function f, the double integral determines **the volume below the graph of** $f(x,y)$ **and above the** xy**-plane**.

- Section 14.2: *Double Integrals over General Regions.*

  $\int\int_R f(x,y)\ dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\ dydx$ over a nonrectangular region R (for each x between a and b, y runs from $g_1(x)$ to $g_2(x)$) or $\int\int_R f(x,y)\ dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\ dxdy$ for appropriate choice of h_1, h_2, c, d (for each y between c and d, x runs from $h_1(y)$ to $h_2(y)$). This is Fubini's Theorem in Stronger Form. As in 14.1, for a positive function f, the double integral determines **the volume below the graph of** $f(x,y)$ **and above the** xy**-plane**.

- Section 14.3: *Area and Average Value of a Function by Double Integration.*

  - If $f(x,y) = 1$, $\int\int_R dA$ gives **the area of the region** R.
  - **The average value** of $f(x,y)$ over R is $\frac{1}{\text{area of } R} \int\int_R f(x,y)\ dA$.

- Section 14.4: *Double Integrals in Polar Form.*

  Know how to set up and do double integrals in polar coordinates. Use them when appropriate!
  Don't forget "r": $dA = rdrd\theta$ now! That is,
  $\int\int_R f(x,y)\ dA = \int\int_G f(r\cos\theta, r\sin\theta) rdrd\theta$ (where G is the same as R, but now described in polar coordinates).
  Setting $f = 1$ in the integral gives **the area of the region in polar form:** $\int\int_G rdrd\theta$.

- Section 14.5: *Triple Integrals in Rectangular Coordinates.*

  Know how to *set up* a triple integral $\int\int\int_D f(x,y,z)\ dV$ in any variable order and set up and *do* the integral in $dzdydx$ order.
  $\int\int\int_D dV$ (where $f(x,y,z) = 1$) gives **the volume of the region** D.
  The average value of $f(x,y,z)$ over D is $\frac{1}{\text{volume of } D} \int\int\int_D f(x,y,z)\ dV$.

- Section 14.6: *Mass, First Moments, and Center of Mass.*

  Know how to find first moments, mass, and center of mass of either two- or three-dimensional objects (*ignore moments of inertia*).
  Do not forget about symmetry: if the object is symmetric about one of the axes, then you need to find the coordinate of the center of mass on the axis of symmetry only, the other two coordinates will be zero.

- Section 14.7: *Triple Integrals in Cylindrical and Spherical Coordinates.*

  Know how to set up and do a triple integral $\int\int\int_D f\ dV$ in these coordinate systems:

  - cylindrical (r,θ,z): $\int\int\int_D f\ dV = \int\int\int_D f(r,\theta,z) dz r dr d\theta$ (**do not forget** r **here!**)
  - spherical (ρ,ϕ,θ): $\int\int\int_D f\ dV = \int\int\int_D f(\rho,\phi,\theta)\rho^2 \sin\phi\ d\rho d\phi d\theta$ (**here you need** $\rho^2 \sin\phi$ in the integrand).

  Use the coordinate conversion formulas!

- Sections 15.1: *Line Integrals.*

  Know how to set up and evaluate line integrals $\int_C f\ ds$ over parameterized curves in 2D or 3D. Remember the additivity property for piecewise curves.