

## MTH 212 - MULTIVARIATE CALCULUS - STUDY GUIDE FOR EXAM I

**No books, notes, calculators, or cell phones are permitted during the test!**

**Use notes, text, homework, and suggested exercises to prepare for the test.**

- Chapter 11: “Vectors and the Geometry of Space”.

- Section 11.1: *3D Coordinate System*.

- \* Three-dimensional Cartesian (rectangular) coordinate system.
- \* Finding distance between two points  $P_1(x_1, y_1, z_1)$  and  $P_2(x_2, y_2, z_2)$  by  $|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ .
- \* The standard equation for the sphere of radius  $a$  and center  $(x_0, y_0, z_0)$ :  $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$ . Equations of coordinate planes and planes parallel to them, circular cylinders, etc.

- Section 11.2: *Vectors*.

- \* Basics: definition of a vector, component form of a vector, magnitude (length) of a vector, addition (difference) and multiplication by a scalar, properties of vector operations, unit vectors, standard unit vectors, midpoint of a line segment.

- Section 11.3: *The Dot Product, Vector Projections, Work*.

- \* The dot product formulas:  $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 = |\mathbf{u}||\mathbf{v}| \cos \theta$ . (Also: review the properties of the dot product.)
- \* Angle between two vectors:  $\theta = \cos^{-1} \left( \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|} \right)$ .
- \* Vector projection:  $\text{proj}_{\mathbf{v}} \mathbf{u} = \left( \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|} \right) \left( \frac{\mathbf{v}}{|\mathbf{v}|} \right) = \left( \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2} \right) \mathbf{v}$ . (The scalar component of  $\mathbf{u}$  in the direction of  $\mathbf{v}$  is  $\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|}$ .)
- \* Work done by a constant force  $\mathbf{F}$  in the direction of displacement  $\mathbf{D}$  is  $W = \mathbf{F} \cdot \mathbf{D}$ .

- Section 11.4: *The Cross Product*.

- \* The cross product formulas:  $\mathbf{u} \times \mathbf{v} = (|\mathbf{u}||\mathbf{v}| \sin \theta) \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = (u_2v_3 - v_2u_3)\mathbf{i} - (u_1v_3 - v_1u_3)\mathbf{j} + (u_1v_2 - v_1u_2)\mathbf{k}$ . (Also: review the properties of the cross product.)
- \* Areas of a triangle  $((1/2)|\mathbf{u} \times \mathbf{v}|)$  and parallelogram  $(|\mathbf{u} \times \mathbf{v}|)$  determined by vectors  $\mathbf{u}$  and  $\mathbf{v}$ .
- \* Volume of a parallelepiped determined by vectors  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  via triple scalar (or box) product:  $|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|$ , where  $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$ .

- Section 11.5: *Lines and Planes in Space*.

- \* Vector and parametric equations for a line in space (need a point on the line,  $P_0(x_0, y_0, z_0)$ , and a vector  $\mathbf{v}$  the line is parallel to). Note: parametrizations are not unique!

- \* Distance from point  $S$  to a line passing through point  $P$ , parallel to  $\mathbf{v}$ :  $d = \frac{|\mathbf{PS} \times \mathbf{v}|}{|\mathbf{v}|}$ . (If the line is given to you in a parametric form, you can find point  $P$  on the line by plugging in any value of  $t$ , e.g.,  $t = 0$ .)
- \* For a plane in space: need a point on the plane,  $P_0(x_0, y_0, z_0)$ , and a vector normal (i.e., orthogonal) to the plane,  $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$ . The equation is  $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$ . (One way to get the normal  $\mathbf{n}$  is take the cross product of two vectors in the plane (that have the same initial point).)
- \* Distance from a point to a plane: given point  $S$  in space, and a plane with normal  $\mathbf{n}$  and point  $P$  on the plane, the distance from  $S$  to the plane is  $d = \frac{|\mathbf{PS} \cdot \mathbf{n}|}{|\mathbf{n}|}$ .
- \* Line of intersection of two planes: given two planes with normal vectors  $\mathbf{n}_1$  and  $\mathbf{n}_2$ , respectively, the vector  $\mathbf{n}_1 \times \mathbf{n}_2$  points in the direction of the line of intersection of the two planes (assuming they intersect). To get a point on this line, you can solve the system of two plane equations. Note that there will be 3 variables and 2 equations in this linear system, so you should just set one of the variables to a constant, e.g.,  $x = 0$ , and solve for the other two. (Two planes are parallel – don't intersect – if they have parallel normals, i.e.,  $\mathbf{n}_1 = k\mathbf{n}_2$  .)
- \* Review other examples from class about relations between lines and planes: parallel, skew, and intersecting lines, intersecting line and plane, etc.
- \* The angle between two planes is the angle between their normal vectors.