MTH 212 - MULTIVARIATE CALCULUS - STUDY GUIDE FOR EXAM II

No books, notes, calculators, or cell phones are permitted during the test!

Use notes, text, homework, and suggested exercises to prepare for the test.

- Chapter 12: "Vector Functions and Motion in Space".
  - Sections 12.1: *Position, velocity, and acceleration.*
    - Parametric curves on the plane and in space.
    - For a vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$, discuss its domain and continuity, and find its limit as t approaches some t_0.
    - If a vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$ represents the position of a particle moving along a curve, then $\mathbf{v}(t) = \mathbf{r}'(t)$ is the velocity, $|\mathbf{v}(t)|$ is the speed, and $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$ is the acceleration.
    - Review the differentiation rules for vector functions.
    - Fact: if $\mathbf{r}(t)$ is a differentiable vector function of constant length ($|\mathbf{r}| = c$), then $\mathbf{r} \cdot \mathbf{r}' = 0$.
  - Section 12.2: *Integrals of Vector Functions.*
    - Know how to integrate vector functions and how to solve initial value problems (for examples, if you are given $\mathbf{a}(t)$ and initial values of $\mathbf{v}(t_0)$ and $\mathbf{r}(t_0)$ at some point t_0, then you need to integrate to work your way up to $\mathbf{r}(t)$, using the values of $\mathbf{v}(t_0)$ and $\mathbf{r}(t_0)$ to find the constants of integration).
  - Section 12.3: *Arc length in Space* (similar formula for plane curves). *Arc Length Parameter.*
    - If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ gives a curve in space, the arc length from the point at time $t = a$ to the point at time $t = b$ is

$$\int_a^b |\mathbf{v}(t)|\, dt = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt.$$

      Recall that the arc length parameter $s(t) = \int_a^t |\mathbf{v}(\tau)|\, d\tau$ can be used to reparameterize the curve.
  - Sections 12.4 and 12.5: *Curvature, Unit Tangent, Normal, and Binormal Vectors, Decomposition of Acceleration.*
    - $\mathbf{T} = \mathbf{v}/|\mathbf{v}|$ (unit tangent vector)
    - $\mathbf{N} = (d\mathbf{T}/dt)/|d\mathbf{T}/dt|$ (principal unit normal vector)
    - $\mathbf{B} = \mathbf{T} \times \mathbf{N}$ (binormal vector)
    - $\kappa = (1/|\mathbf{v}|)|d\mathbf{T}/dt|$ (curve curvature).
    - Acceleration decomposed: $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$, where $a_T = \frac{d}{dt}(|\mathbf{v}|)$ and $a_N = \sqrt{|\mathbf{a}|^2 - a_T^2}$ are the tangential and normal components of acceleration, respectively.

- From Chapter 10:
  - Section 10.3: *Polar Coordinates.*
    * Definition of polar coordinates (r, θ).
    * Equations in polar coordinates; converting points and equations between polar and Cartesian coordinates using $x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$, $\tan\theta = y/x$.

- Chapter 13: "Partial Derivatives".
  - Section 13.1 and 13.2: *Functions of Several Variables. Limits and Continuity.*
    * Domain and range of functions of two and three variables, $z = f(x, y)$ and $w = f(x, y, z)$.
    * Graph of $f(x, y)$ ($z = f(x, y)$ gives a surface in space), level curves $f(x, y) = c$ in the xy-plane and level surfaces $f(x, y, z) = c$ in space.
      Recall quadric surfaces in Section 11.6 - it's convenient to recognize spheres, ellipsoids, paraboloids, etc. as graphs or level surfaces of some functions.
    * Basic limit computations. Two-path test for non-existence of a limit.
    * Continuity: $f(x, y)$ is continuous at a point (x_0, y_0) if $\lim_{(x,y)\to(x_0,y_0)} f(x, y) = f(x_0, y_0)$.
  - Section 13.3: *Partial Derivatives.*
    * Partial derivatives f_x, f_y (and f_z, if f is a function of x, y, z). If f_x and f_y are continuous at (a, b) then $f(x, y)$ is differentiable, and, therefore, continuous at (a, b).
    * Mixed Derivatives: for $f(x, y)$ continuous together with its partial derivatives at a point (a, b), $f_{xy}(a, b) = f_{yx}(a, b)$.