# MTH 212 - MULTIVARIATE CALCULUS - STUDY GUIDE FOR EXAM IV

*No books, notes, calculators, or cell phones are permitted during the test!*

*Use notes, text, homework, and suggested exercises to prepare for the test.*

Chapter 14 "Multiple Integrals"

- Section 14.4: *Double Integrals in Polar Form.*

  Know how to set up and evaluate double integrals in polar coordinates. Use them when appropriate. **Don't forget "$r$":** $dA = rdrd\theta$ now! That is,
  $\int\int_R f(x,y)\ dA = \int\int_G f(r\cos\theta, r\sin\theta) rdrd\theta$ (where $G$ is the same as $R$, but now described in polar coordinates).
  As above, setting $f = 1$ in the integral gives *the area of the region in polar form:* $\int\int_G rdrd\theta$.

  Section 14.5: *Triple Integrals in Rectangular Coordinates.*

  Know how to set up a triple integral $\int\int\int_D f(x,y,z)\ dV$ in any variable order and set up and evaluate the integral in $dzdydx$ order.
  $\int\int\int_D dV$ (where $f(x,y,z) = 1$) gives **the volume of the region** $D$.
  **The average value** of $f(x,y,z)$ over $D$ is $\frac{1}{\text{volume of } D}\int\int\int_D f(x,y,z)\ dV$.

- Section 14.6: *Mass, First Moments, and Center of Mass.*

  Know how to find first moments, mass, and center of mass of either two- or three-dimensional objects (*ignore moments of inertia*).
  Do not forget about symmetry: if the object is symmetric about one of the axes, then you need to find the coordinate of the center of mass on the axis of symmetry only, the other coordinates will be zero.

- Section 14.7: *Triple Integrals in Cylindrical and Spherical Coordinates.*

  Know how to set up and do a triple integral $\int\int\int_D f\ dV$ in these coordinate systems:

  - cylindrical $(r,\theta,z)$: $\int\int\int_D f\ dV = \int\int\int_D f(r,\theta,z)\ dzrdrd\theta$ (**do not forget** $r$ **here**)
  - spherical $(\rho,\phi,\theta)$: $\int\int\int_D f\ dV = \int\int\int_D f(\rho,\phi,\theta)\rho^2\sin\phi\ d\rho d\phi d\theta$ (**here you need** $\rho^2\sin\phi$ **in the integrand**).

  Use the coordinate conversion formulas!

- Section 14.8: *Substitution in Double Integrals.*

  Know how to do substitutions in double integrals. Use the Jacobian $J(u,v)$ for a coordinate transformation $x = g(u,v)$ and $y = h(u,v)$. When setting up the new integral, use **the absolute value of** $J(u,v)$ in the integrand. Know how to figure out the new bounds of the region from the old ones.