MTH 212 - MULTIVARIATE CALCULUS - STUDY GUIDE FOR EXAM III

No books, notes, calculators, or cell phones are permitted during the test!

Use notes, text, homework, and suggested exercises to prepare for the test.

You will be required to use the lockdown browser on live.wilkes.edu and a scanning app (or scanner) to upload your written exam solutions to a folder on live.wilkes.edu within 20 minutes right after the one-hour test.

Chapter 13: "Partial Derivatives".

- Section 13.5: *Directional Derivatives and Gradient Vectors.*
  - The derivative of the function f in the direction of a unit vector $\mathbf{u}$ at the point P_0 is $D_{\mathbf{u}}f(P_0) = \nabla f(P_0) \cdot \mathbf{u}$. (Make sure $\mathbf{u}$ is a unit vector, if not, divide it by the length and then use in the formula for $D_{\mathbf{u}}f(P_0)$.) ∇f is the gradient vector of f (the vector of partial derivatives). - *Recall: this part was on Exam II, but you need to know it for Exam III too.*
  - The direction of the greatest increase of f is $\mathbf{u} = \nabla f$ and $D_{\mathbf{u}}f(P_0) = |\nabla f(P_0)|$. The direction of the greatest decrease of f is $\mathbf{u} = -\nabla f$ and $D_{\mathbf{u}}f(P_0) = -|\nabla f(P_0)|$. The function f has no change in the direction $\mathbf{u}$ normal to ∇f, i.e. $D_{\mathbf{u}}f(P_0) = 0$.
  - At every point in the domain of f, the gradient ∇f is normal to the level curve going through the point. Based on this fact, the tangent line to a level curve $f(x, y) = c$ at a point $P_0(x_0, y_0)$ is given by $f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) = 0$.

- Section 13.6 *Tangent Planes.*
  - The equation of the plane tangent to the level surface $f(x, y, z) = c$ at a point $P_0(x_0, y_0, z_0)$ is $f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0) = 0$.
    Normal line to the surface at the point P_0 has parametric equations $x = x_0 + f_x(P_0)t$, $y = y_0 + f_y(P_0)t$, $z = z_0 + f_z(P_0)t$.
  - The equation of the plane tangent to the surface given by $z = f(x, y)$ at a point $P_0(x_0, y_0, z_0)$ is $f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) - (z - z_0) = 0$ where $z_0 = f(x_0, y_0)$.
  - The linearization of $f(x, y)$ at a point (x_0, y_0) is $L(x, y) = z_0 + f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0)$ where $z_0 = f(P_0)$.

- Section 13.7 *Extreme Values and Saddle Points.*
  - To find the critical points of $f(x, y)$, set all partials to 0, $f_x = f_y = 0$, find all solutions and check the points where f_x, f_y do not exist. For each critical point, compute the Hessian $H = f_{xx}f_{yy} - f_{xy}^2$. If $H < 0$ then this is a saddle point. For $H > 0$, if $f_{xx} > 0$ then this is a local minimum and if $f_{xx} < 0$ then this is a local maximum. If $H = 0$, the test is inconclusive.

- Section 13.8 *Lagrange multipliers (problems with equality constraints).*
  - For problems with one equality constraint:
    Determine an objective function f and constraint function g. Solve the system of equations

    $$\nabla f = \lambda \nabla g$$
    $$g = 0$$

    for x, y, and λ. Evaluate f at the solution points and find the max and min values of f among these values. (Solving the system can be tricky. The best thing you can do to prepare is to try lots of problems.)

    For problems with two equality constraints, you have to know how to set up a system of equations (you will not be asked to solve the system).

Chapter 14 "Multiple Integrals".

- Section 14.1: *Double and Iterated Integrals over Rectangles.*

  Evaluate a double integral of f over a rectangular area R, $\int\int_R f(x,y)\, dA$, using the iterated (repeated) integral $\int_a^b \int_c^d f(x,y)\, dydx$ or $\int_c^d \int_a^b f(x,y)\, dxdy$ (First Form of Fubini's Theorem). *For a positive function f, the double integral determines the volume below the graph of $f(x,y)$ and above the xy-plane over R.*

- Section 14.2: *Double Integrals over General Regions.*

  $\int\int_R f(x,y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dydx$ over a nonrectangular region R (for each x between a and b, y runs from $g_1(x)$ to $g_2(x)$) or $\int\int_R f(x,y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dxdy$ (for each y between c and d, x runs from $h_1(y)$ to $h_2(y)$). This is Fubini's Theorem in Stronger Form. As in 14.1, *for a positive function f, the double integral determines the volume below the graph of $f(x,y)$ and above the xy-plane over R.*

- Section 14.3: *Area and Average Value of a Function by Double Integration.*
  - If $f(x,y) = 1$, $\int\int_R dA$ gives *the area of the region R.*
  - *The average value* of $f(x,y)$ over R is $\frac{1}{\text{area of } R} \int\int_R f(x,y)\, dA$.