MTH 212 - MULTIVARIATE CALCULUS - STUDY GUIDE FOR EXAM I

No books, notes, calculators, or cell phones are permitted during the test!
Use notes, text, homework, and suggested exercises to prepare for the test.

- Chapter 11: "Vectors and the Geometry of Space".
  - Section 11.1: *3D Coordinate System.*
    * Three-dimensional Cartesian/rectangular coordinate system.
    * Finding distance between two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ by $|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
    * The standard equation for the sphere of radius a and center (x_0, y_0, z_0): $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$.
  - Section 11.2: *Vectors.*
    * Basics: definition of a vector, component form of a vector, magnitude (length) of a vector, addition (difference) and multiplication by a scalar, properties of vector operations, unit vectors, standard unit vectors, midpoint of a line segment.
  - Section 11.3: *The Dot Product, Vector Projections, Work.*
    * The dot product formulas: $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 = |\mathbf{u}||\mathbf{v}| \cos\theta$. (Also: review the properties of the dot product.)
    * Angle between two vectors: $\theta = \cos^{-1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$.
    * Vector projection: $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}\right)\left(\frac{\mathbf{v}}{|\mathbf{v}|}\right) = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$. (The scalar component of $\mathbf{u}$ in the direction of $\mathbf{v}$ is $\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}$.)
    * Work done by a constant force $\mathbf{F}$ in the direction of displacement $\mathbf{D}$ is $W = \mathbf{F} \cdot \mathbf{D}$.
  - Section 11.4: *The Cross Product.*
    * The cross product formulas: $\mathbf{u} \times \mathbf{v} = (|\mathbf{u}||\mathbf{v}| \sin\theta)\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} =$ $(u_2v_3 - v_2u_3)\mathbf{i} - (u_1v_3 - v_1u_3)\mathbf{j} + (u_1v_2 - v_1u_2)\mathbf{k}$. (Also: review the properties of the cross product.)
    * Areas of a triangle ($(1/2)|\mathbf{u} \times \mathbf{v}|$) and parallelogram ($|\mathbf{u} \times \mathbf{v}|$) determined by vectors $\mathbf{u}$ and $\mathbf{v}$.
    * Volume of a parallelepiped determined by vectors vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ via triple scalar (or box) product: $|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|$, where $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_2 \end{vmatrix}$.
  - Section 11.5: *Lines and Planes in Space.*
    * Vector and parametric equations for a line in space (need a point on the line, $P_0(x_0, y_0, z_0)$, and a vector $\mathbf{v}$ the line is parallel to). Note: parameterizations are not unique!

  - Distance from point S to a line passing through point P, parallel to $\mathbf{v}$: $d = \frac{|\mathbf{PS} \times \mathbf{v}|}{|\mathbf{v}|}$. (If the line is given to you in a parametric form, you can find point P on the line by plugging in any value of t, e.g., $t = 0$.)
  - For a plane in space: need a point on the plane, $P_0(x_0, y_0, z_0)$, and a vector normal (i.e., orthogonal) to the plane, $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$. The equation is $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$. (One way to get the normal $\mathbf{n}$ is take the cross product of two vectors in the plane (that have the same initial point).)
  - Distance from a point to a plane: given point S in space, and a plane with normal $\mathbf{n}$ and point P on the plane, the distance from S to the plane is $d = \frac{|\mathbf{PS} \cdot \mathbf{n}|}{|\mathbf{n}|}$.
  - Line of intersection of two planes: given two planes with normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$, respectively, the vector $\mathbf{n}_1 \times \mathbf{n}_2$ points in the direction of the line of intersection of the two planes (assuming they intersect). To get a point on this line, you can solve the system of two plane equations. Note that there will be 3 variables and 2 equations in this linear system, so you should just set one of the variables to a constant, e.g., $x = 0$, and solve for the other two. (Two planes are parallel – don't intersect – if they have parallel normals, i.e., $\mathbf{n}_1 = k\mathbf{n}_2$.)
  - The angle between two planes is the angle between their normal vectors.

- Chapter 12: "Vector Functions and Motion in Space".
  - Sections 12.1: *Position, velocity, and acceleration.*
    - Parametric curves on the plane and in space.
    - For a vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, discuss its domain and continuity, and find its limit as t approaches some t_0.
    - If a vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ represents the position of a particle moving along a curve, then $\mathbf{v}(t) = \mathbf{r}'(t)$ is the velocity, $|\mathbf{v}(t)|$ is speed, and $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$ is the acceleration.