MTH/CS 364/464 Numerical Analysis
EXAM 2
Due Monday (in class), April 15th, 2019

Instructions: This is a take-home exam. You may *not* discuss the exam problems with anyone but me, the work should be yours only. Turn in this cover page and all of the work that you have decided to submit. Please write/print your solutions on your own paper, show your work. Start a new page for each problem and staple all the pages.

NAME: ________________________________	**SCORE:** ________

1. (12pts) A complete cubic spline s for a function f is defined on the interval $[1, 3]$ by

$$s(x) = \begin{cases} s_0(x) = 3(x-1) + 2(x-1)^2 + (x-1)^3, & 1 \leq x \leq 2, \\ s_1(x) = a + b(x-2) + c(x-2)^2 + d(x-2)^3, & 2 \leq x \leq 3. \end{cases}$$

   (Note $s_0'(1) = f'(1)$ and $s_1'(3) = f'(3)$ must hold for this spline.) Given $f'(1) = f'(3)$, find a, b, c, and d.

2. (12pts) Consider a forward-difference approximation for the second derivative of the form

$$f''(x) \approx Af(x) + Bf(x+h) + Cf(x+2h).$$

   Use Taylor's theorem for $f(x+h)$ and $f(x+2h)$ to determine the coefficients A, B, and C that give the maximal possible order of accuracy in terms of h.

3. (12pts) Derive the Newton-Cotes (quadrature) formula for $\int_0^1 f(x)\, dx$, that is $\int_0^1 f(x)\, dx \approx \sum_{i=0}^{4} A_i f(x_i)$ using the nodes $x_0 = 0$, $x_1 = 1/3$, $x_2 = 2/3$, and $x_3 = 1$.

4. (24pts) Write a MATLAB code to approximate

$$\int_0^1 \cos(x^2)\, dx$$

   using the composite trapezoid rule and one to approximate the integral using the composite Simpson's rule, with equally spaced nodes. The number of intervals $n = 1/h$ should be an input to each code. Turn in listings of your codes and the following results:
   Do a convergence study to verify the second order accuracy of the composite trapezoidal rule and the fourth order accuracy of the composite Simpson's rule; that is, run your code with several different h values and make a table showing the error E_h with each value of h and the ratios E_h/h^2 for the composite trapezoidal rule and

E_h/h^4 for the composite Simpson's rule. These ratios should be nearly constant for small values of h. You can determine the error in your computed integral by comparing your results with those of MATLAB routine *quad*. To learn about routine *quad*, type "help quad" in MATLAB. When you run *quad*, ask for a high level of accuracy, say,

$$\text{q} = \text{quad}('\cos(x.\hat{}2)', 0, 1, [1.e-12\ 1.e-12]),$$

where the last argument $[1.e-12\ 1.e-12]$ indicates that you want an answer that is accurate to 10^{12} in both a relative and an absolute sense. (Note that when you use routine *quad* you must define a function, either inline or in a separate file, that evaluates the integrand $\cos(x^2)$ at a vector of values of x; hence you need to write $\cos(x.\hat{}2)$, instead of $\cos(x\hat{}2)$.)

5. (10pts) A car laps a race track in 84 seconds. The speed of the car at each 6-second interval is determined using a radar gun and is given, from the beginning of the lap, in feet per second, by the entries in the following table. Estimate the length of the track by using any two methods from numerical integration.

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

6. (15 pts) Consider the integration formula

$$\int_{-1}^{1} f(x)\,dx \approx f(\alpha) + f(-\alpha).$$

(a) For what value(s) of α, if any, will this formula be exact for all polynomials of degree 1 or less?

(b) For what value(s) of α, if any, will this formula be exact for all polynomials of degree 3 or less?

(c) For what value(s) of α, if any, will this formula be exact for all polynomials of the form $a + bx + cx^3 + dx^4$?

7. (15 pts) *Mandatory for MTH 464 student, bonus question for the others:* Approximate the integral using Gauss quadrature with $n = 2$, and compare your result to the exact value. Show your work.

$$\int_{1}^{1.5} x^2 \ln x\,dx.$$

HONOR PLEDGE: I affirm that I have not given, received, or used any unauthorized assistance on this exam.

Your Signature: