Trig integration review cheat sheet

REMEMBER THESE THREE IDENTITIES!

- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$
- $\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$

Then, for a general integral $\int \sin^n(x)\cos^m(x)dx$ where m, n are positive integers:

- If n is **odd**, rewrite $\sin^n(x) = \sin(x)\sin^{n-1}(x)$ and then use the first identity above to rewrite $\sin^{n-1}(x)$ in terms of $\cos(x)$ and use a u-substitution to complete the integration.

  EXAMPLE

$$\int \sin^3(x)\cos^4(x)dx = \int \sin(x)\left(\sin^2(x)\cos^4(x)\right)dx = \int \sin(x)\left[\left(1 - \cos^2(x)\right)\cos^4(x)\right]dx$$

  Now let $u = \cos(x)$ so $du = -\sin(x)dx$. Then the integral becomes

$$\int -[(1-u^2)u^4]du = -\int u^4 - u^6 du = -(1/5u^5 - 1/7u^7) + C = -1/5\cos^5(x) + 1/7\cos^7(x) + C.$$

  NOTE: The same process works if instead m is odd.

- If **both** n and m are **even**, then use the identities above to change the integrand to be in terms of $\cos(2x)$.

  EXAMPLE

$$\begin{aligned}
\int \sin^4(x)\cos^2(x)dx &= \int \left[\frac{1}{2}(1-\cos(2x))\right]^2\left[\frac{1}{2}(1+\cos(2x))\right]dx \\
&= \frac{1}{8}\int (1 - 2\cos(2x) + \cos^2(2x))(1+\cos(2x))dx \\
&= \frac{1}{8}\int 1 - \cos(2x) - \cos^2(2x) + \cos^3(2x)dx \\
&= \frac{1}{8}\int 1 - \cos(2x) - \frac{1}{2}(1+\cos(4x)) + \cos(2x)(1-\sin^2(2x))dx \\
&= \frac{1}{8}\left[\int 1 - \cos(2x) - \frac{1}{2} - \frac{1}{2}\cos(4x)dx + \int \cos(2x)(1-\sin^2(2x))dx\right] \\
&= \frac{1}{8}\left[x - 1/2\sin(2x) - 1/2x - 1/8\sin(4x) + C + \int 1/2(1-u^2)du\right] \\
&= \frac{1}{8}\left[1/2x - 1/2\sin(2x) - 1/8\sin(4x) + 1/2\sin(2x) - 1/6\sin^3(2x)\right] + C \\
&= \frac{1}{8}\left[1/2x - 1/8\sin(4x) - 1/6\sin^3(2x)\right] + C
\end{aligned}$$